The Great Accretion And The Great Depression

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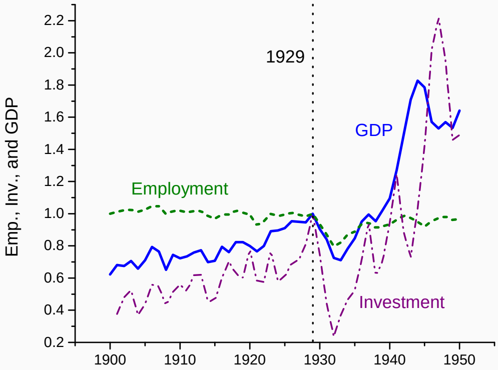
Introduction and Motivation

- **1920s**: Period of unprecedented prosperity (Roaring Twenties, **Great Accretion**).
  - Peak of the Second Industrial Revolution.
  - Electrification, the automobile and the plane, the petrochemical industry, etc..
- This ended in **1929** with the onset of the **Great Depression**.
  - Blamed on bad monetary policy, stock market crashes, tariffs, etc.
  - These mechanisms sound more like *reaction to* and *propagation of* some earlier trigger.

Evidence - GDP, Employment and Investment

- Strong investment and GDP growth up to 1929.
- Flat (or mildly declining) employment during the 1920s.
- Massive crash during the Great Depression.
  - output down by 29%, employment by 18% in 1933.



Our Paper

Integrated theory of the 1920s and early ’30s:

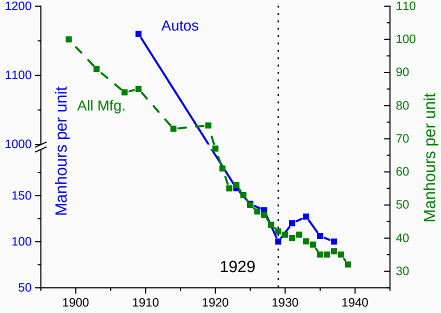
- **1920s**:
  - process innovation starts to far outpace product innovation.
  - expectation of continued product innovation and demand $\Longrightarrow$ over-accumulation of capital.
- **1929**:
  - realization that product innovation has stalled $\Longrightarrow$ satiation in demand.
  - continued process innovation $\Longrightarrow$ lower demand for labor.

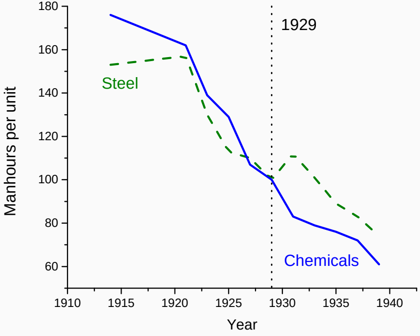
Research Question

Could the dynamics of the 1920s have *contributed* to causing the Great Depression?

Evidence - Process Innovation

- Rapid decline in the number of manhours required to produce a car.
  - Electrification and the assembly line provide a huge boost.
- Trend common to all manufacturing and continuing up to 1929 and beyond.

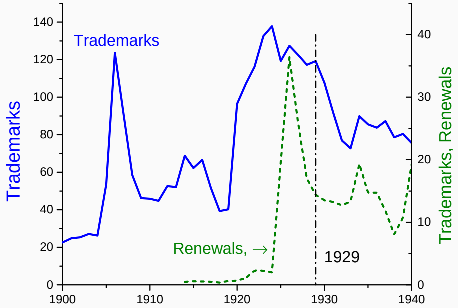


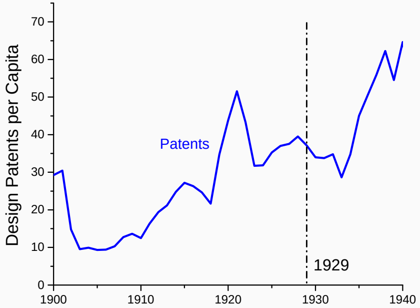


Evidence - Product Innovation

Full Series

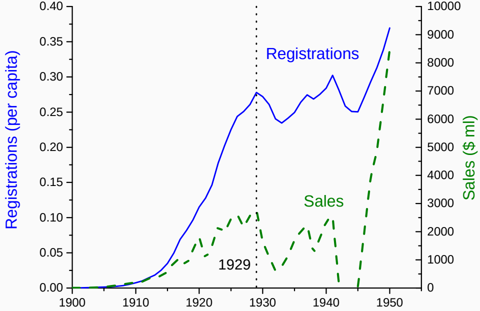
- Burst of trademark and patent activity in the first two decades of the 1900s.
- Trends stall around the middle of the decade.

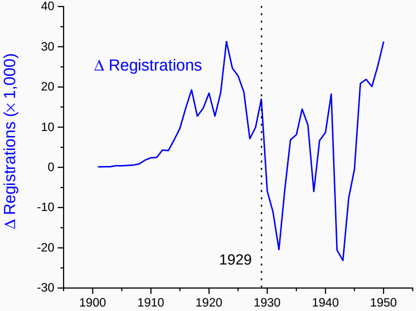




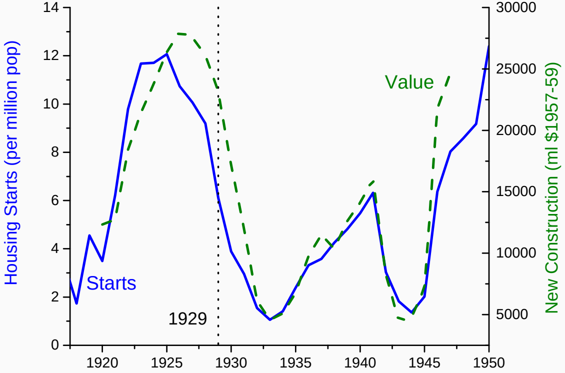
Evidence - Demand Saturation (1/2)

- Demand for durables like autos and housing peaks during the late 1920s.





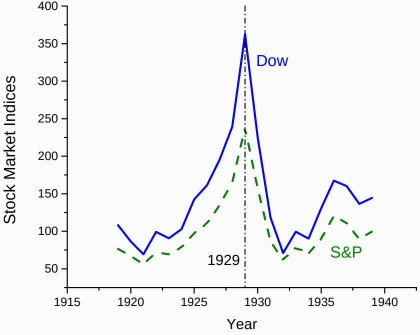
Evidence - Demand Saturation (2/2)

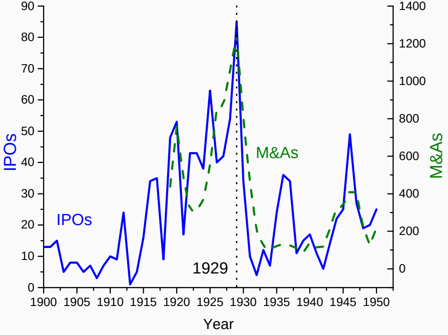


Evidence - Business Optimism

- Huge run-up in stock market until 1929.
  - We can think of it as a proxy for people's expectations about future growth.
- IPOs and M&As paint a similar picture.

Profits





Model - Outline

- Modeled after *Yorukoglu [2000]*.
- On the surface, standard monopolistic competition.
- **Two sources of technological progress** (fully exogenous):
    - labor productivity growth (*process innovation*);
    - expansion of varieties (*product innovation*).
- **Key features** of the model:
    - *extensive margin*: upper bound on available number of varieties.
    - *intensive margin*: lower bound on consumption of each variety.

Model - Household

- Standard preferences with Dixit-Stiglitz aggregator and endogenous labor

$$U\left(\left\{\{c_{jt}\}_{j=0}^{N}, N_t, l_t\right\}_t\right) = \max_{\substack{c_{jt}\in\{0,[\mathfrak{c},\infty)\},\\ N_t\leqslant\mathfrak{N}_t}} \sum_{t=0}^{\infty}\beta^t\left[\alpha\log\left(\int_0^{\mathfrak{N}_t} c_{jt}^{\theta}\,dj\right)^{\frac{1}{\theta}} - (1-\alpha)\frac{l_t^{1+\chi}}{1+\chi}\right], \quad \theta<1,$$

  with:

  - **minimum consumption level**:

$$c_{jt}\in\{0,[\mathfrak{c},\infty)\};$$

  - **upper bound on available varieties**:

$$N_t\leqslant\mathfrak{N}_t.$$

- Capital is the only means of saving:

$$\int_0^N p_{jt}c_{jt}\,di + [k_{t+1}-(1-\delta)k_t] = w_t l_t + r_t k_t + \Pi_t.$$

Model - Household's FOCs (1/2)

- Symmetric equilibrium: $c_{jt} = c_t$.
- The FOCs are

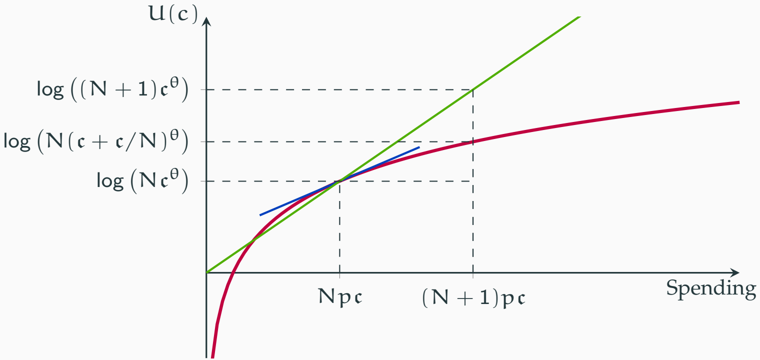
$$\left[\frac{\partial}{\partial N_t}\right]: \quad \frac{1}{\theta}\frac{\alpha}{N_t} \geqslant \lambda_t p_t c_t \qquad \text{(\textbf{Extensive Margin})}$$

$$\left[\frac{\partial}{\partial c_t}\right]: \quad \frac{\alpha}{N_t} \leqslant \lambda_t p_t c_t \qquad \text{(\textbf{Intensive Margin})}$$

- Since $\theta < 1$, both cannot hold at the same time with equality!

Model - Household's FOCs (2/2)

- Consider Nc^{θ}, the subutility associated with $\ln\left(Nc^{\theta}\right)^{1/\theta}$, where $\theta < 1$.
- The cost of adding an extra variety is $p\mathfrak{c}$.
- Because of **concavity** of the utility function, $u(\mathfrak{c}) > u'(\mathfrak{c})\mathfrak{c}$.



Model - Production of a Variety

- Linear production function in variety-specific intermediate:

$$o_{jt} = m_{jt}.$$

- Price of the intermediate good is normalized to 1.
- Produced by a **monopolist** who charges price p_{jt} to maximize:

$$\underbrace{\pi_{jt}}_{\textbf{Profits}} = \underbrace{D_{jt}(\mathbf{p}_t)\, p_{jt}}_{\textbf{Total Revenue}} - \underbrace{m_{jt}}_{\textbf{Production Costs}}.$$

Model - Zone 1: Perfect Competition

Zone 1 (*Extensive Margin*):

$$\left[\frac{\partial}{\partial N_t}\right]: \quad \frac{1}{\theta}\frac{\alpha}{\mathbf{N_t}} = \lambda_t p_t \mathfrak{c} \qquad \text{(Interior, Extensive Margin)}$$

$$\left[\frac{\partial}{\partial c_t}\right]: \quad \frac{\alpha}{\mathbf{N_t}} < \lambda_t p_t \mathfrak{c} \qquad \text{(Corner, Intensive Margin)}$$

- consumption is at the lower bound for all varieties that get consumed:

$$c = \mathfrak{c}, N \leqslant \mathfrak{N};$$

- goods are perfect substitutes and the agent consumes a random subset;
- final good producers are **perfectly competitive**, hence $p = 1$.

Graph Labor Condition

Model - Zone 2: Inelastic Demand

Zone 2 (*Shackled Margins*) :

$$\left[\frac{\partial}{\partial N_t}\right]: \quad \frac{1}{\theta}\frac{\alpha}{\mathfrak{N}} = \lambda_t \mathbf{p_t} \mathfrak{c} \qquad \text{(Interior, Extensive Margin)}$$

$$\left[\frac{\partial}{\partial c_t}\right]: \quad \frac{\alpha}{\mathfrak{N}} < \lambda_t \mathbf{p_t} \mathfrak{c} \qquad \text{(Corner, Intensive Margin)}$$

- household consumes all varieties at the lower bound: final output is pinned down

$$N = \mathfrak{N}, c = \mathfrak{c};$$

- at $p = 1$, $\partial/\partial N > 0$ and the upper bound on varieties binds;
- the agent is willing to pay a higher price for each good:

Graph Labor Condition

$$p = \frac{\text{marginal utility of an extra variety}}{\text{marginal utility of income}} > 1$$

Model - Zone 3: Monopolistic Competition

Zone 3 (*Intensive Margin*):

$$\left[\frac{\partial}{\partial N_t}\right]: \quad \frac{1}{\theta}\frac{\alpha}{\mathfrak{N}} > \lambda_t p_t c_t \qquad \text{(Corner, Extensive Margin)}$$

$$\left[\frac{\partial}{\partial c_t}\right]: \quad \frac{\alpha}{\mathfrak{N}} = \lambda_t p_t c_t \qquad \text{(\textbf{Interior, Intensive Margin})}$$

- household consumes all the varieties:

$$N = \mathfrak{N}, c \geqslant \mathfrak{c};$$

- final good producers are **monopolists** and charge fixed mark-up:

$$p = 1/\theta.$$

Graph Labor Condition

Model - Intermediate Good

- Used to produce final varieties and capital.
- Produced by competitive firms using Cobb-Douglas production function:

$$m_t = k_t^{\gamma} \left(\boldsymbol{z_t} l_t \right)^{1-\gamma}.$$

Process innovation works through growth in $\boldsymbol{z_t}$.

- Market clearing condition:

$$\underbrace{k_t^{\gamma} \left(z_t l_t \right)^{1-\gamma}}_{\textbf{Output}} = \underbrace{\int m_{jt} dj}_{\textbf{Final Varieties Production}} + \underbrace{\left[k_{t+1} - (1-\delta) k_t \right]}_{\textbf{Investment}}.$$

Model - Balanced Growth Path

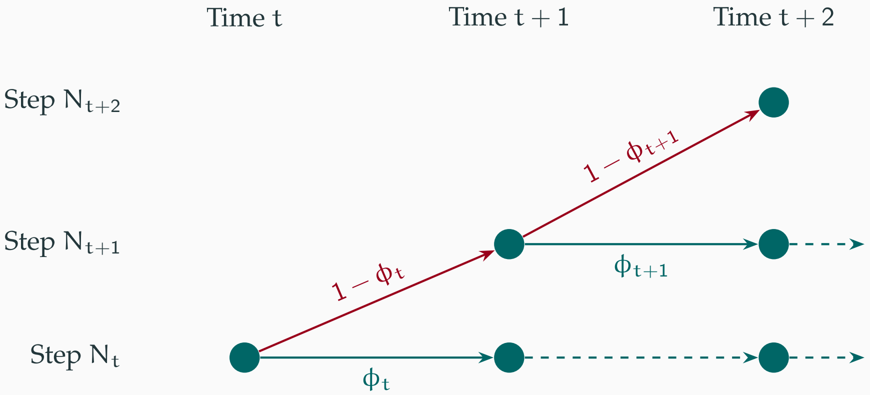
Let g_z denote the growth rate of labor productivity, $g_{\mathfrak{N}}$ the growth rate of available varieties. Assume $\mathfrak{c}$ does not grow.

Result

- ***Zone 1 (Extensive Margin)*** If $g_{\mathfrak{N}} \geqslant g_z \geqslant 1$, there exists a BGP where all aggregate variables grow at rate g_z, $c_t = \mathfrak{c}$, and $N_t \leqslant \mathfrak{N}_t$.
- ***Zone 2 (Shackled Margins)*** If $g_{\mathfrak{N}} = g_z \geqslant 1$, there exists a BGP where all aggregate variables grow at rate g_z, $c_t = \mathfrak{c}$, and $N_t = \mathfrak{N}_t$.
- ***Zone 3 (Intensive Margin)*** If $g_z \geqslant g_{\mathfrak{N}} \geqslant 1$, there exists a BGP where all aggregate variables grow at rate g_z, $c_t \geqslant \mathfrak{c}$ grows at rate $g_z/g_{\mathfrak{N}}$, and $N_t = \mathfrak{N}_t$.

Rational Exuberance (1/3)

- Inspired by *Zeira [1999]*.



Simulation - Rational Exuberance (2/3)

Formally:

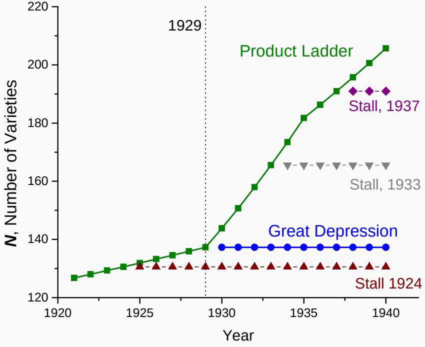
- At any date $t \leqslant T$,
  - product innovation can continue on diagonal path $\{\mathfrak{N}_t^{\uparrow}\}_{t=1}^{T}$.
  - product innovation can stop permanently (**the crash**): stall path $\{\mathfrak{N}_{\bar{t}-1}^{\rightarrow}\}_{t=1}^{T}$.
- $\boldsymbol{\phi_{t+j}^{t}}$: belief formed in period t that a stall will occur in period $t+j$, for $0 \leqslant j \leqslant T-t$.
- $1 - \sum_{j=0}^{T-t} \phi_{t+j}^{t}$: belief that a stall will not occur (with 1 for $t > T$).
- If in period $t+1$ a stall has not occurred yet, the agent updates probabilities using **Bayes' rule**:

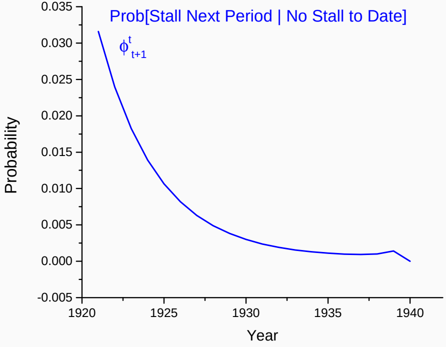
$$\phi_{t+j}^{t+1} = \frac{\phi_{t+j}^{t}}{\sum_{j=1}^{T-j} \phi_{t+j}^{t}} \quad \forall\, 1 \leqslant j \leqslant T-j$$

Simulation - Rational Exuberance (3/3)

- Under diagonal path $\{\mathfrak{N}_t^{\uparrow}\}_{t=1}^{T}$:
  - Slow growth between 1921 and 1929 ($g_{\mathfrak{N}} < g_z$).
  - Strong growth after 1929 to catch up with process gains and converge to zone 1 BGP.
- The economy crashes between 1929 and 1930.

Justification | Simulation Parameters





Simulation - Euler Equation

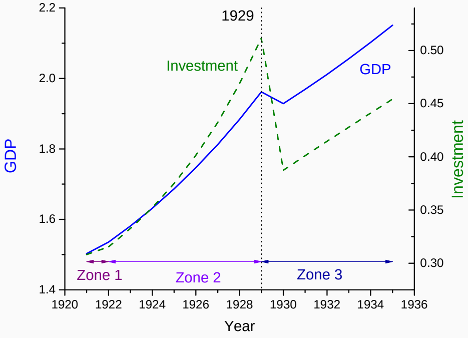
The rational expectations Euler equation reads

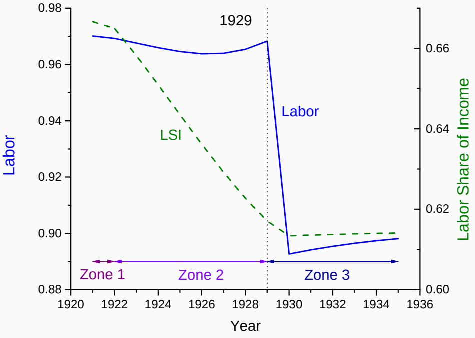
$$\underbrace{\lambda_t^{\uparrow}}_{\textbf{MC}} = \underbrace{\beta\left(1-\phi_{t+1}^{t+1}\right)\lambda_{t+1}^{\uparrow}\left[\gamma k_{t+1}^{\gamma-1}\left(z_{t+1} l_{t+1}^{\uparrow}\right)^{1-\gamma}+(1-\delta)\right]}_{MB_k^{\uparrow}\textbf{: Increase in varieties } (\mathfrak{N}_{t+1} > \mathfrak{N}_t)} +$$

$$\underbrace{\beta\phi_{t+1}^{t+1}\lambda_{t+1}^{\rightarrow}\left[\gamma k_{t+1}^{\gamma-1}\left(z_{t+1} l_{t+1}^{\rightarrow}\right)^{1-\gamma}+(1-\delta)\right]}_{MB_k^{\rightarrow}\textbf{: Permanent stall in varieties } (\mathfrak{N}_{t+1} = \mathfrak{N}_t)}$$

Detailed Algorithm

Simulation - Rational Exuberance: Results





GDP, investment and employment paths.

Full Paths   Other Endogenous Variables

Simulation - Rational Exuberance with Adjustment Costs (1/5)

Given a crash at time τ, **add**:

- **Lost Capital**: fraction "allocated" to $\eta = 1 - \left(\mathfrak{N}_{\tau}^{\rightarrow} / \mathfrak{N}_{\tau}^{\uparrow}\right)$ is destroyed.
- **Adjustment Costs**:
  - **internal**, on variable inputs:

$$A\left(o_{i,\tau}\right) = \frac{\phi}{\kappa}\left(\frac{o_{i,\tau}^{exp}}{1-\eta} - o_{i,\tau}\right)^{-\kappa},$$

where $o_{i,\tau}^{exp} = D\left(\mathbf{p}^{exp}\right)$ was expected demand for period $t = \tau$ at $t-1$.

Simulation - Rational Exuberance with Adjustment Costs (2/5)

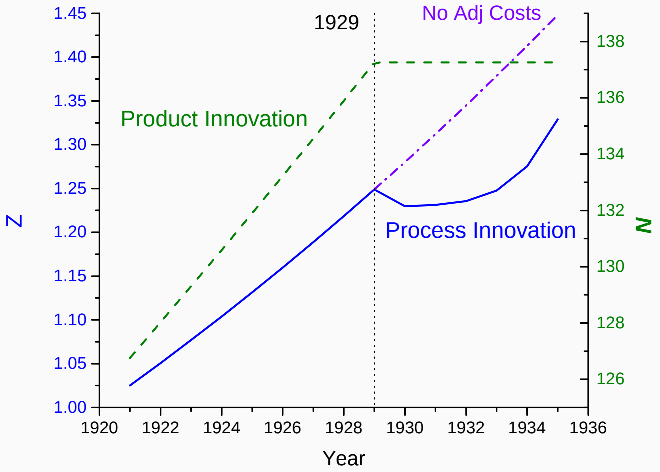
- **external**, on aggregate TFP:

$$z_\tau = \frac{g_z z_{\tau-1}}{\left(\mathbf{o}_\tau^{exp}/\mathbf{o}_\tau\right)^\omega}, \qquad \text{(Impact Effect)}$$

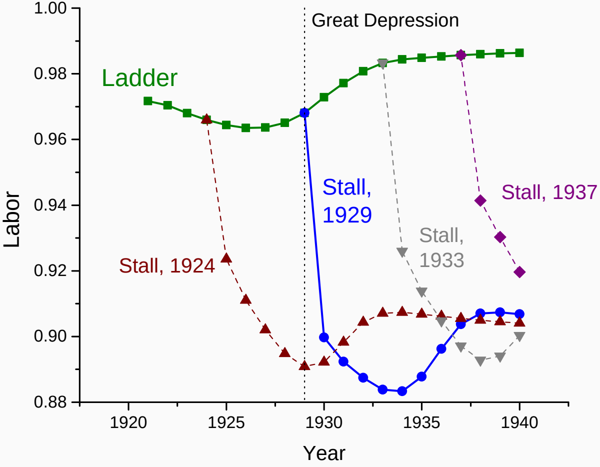
and

$$z_{t+1} = g_z^{t-\tau} z_\tau^{exp} + \underbrace{\frac{1}{1+e^{\rho((t-\tau)-\bar{t})}}}_{\text{Logistic Persistence Parameter}} \left(z_t - g_z^{t-\tau} z_\tau^{exp}\right), \quad t > \tau. \qquad \text{(Propagation)}$$

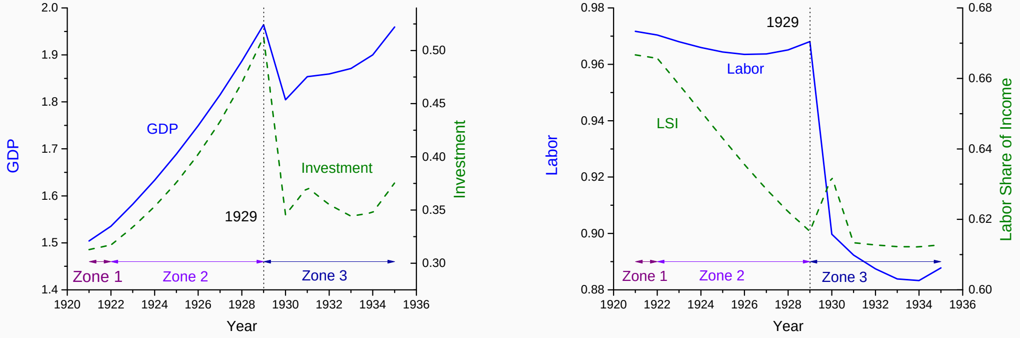
Simulation - Rational Exuberance with Adjustment Costs (3/5)



Simulation - Rational Exuberance with Adjustment Costs (4/5)



Simulation - Rational Exuberance with Adjustment Costs (5/5)



GDP, investment and employment paths with adjustment costs.

Full Paths
Other Endogenous Variables

Conclusions

Main **hypothesis**:

- Product innovation slowed down during the '20s, leading to **demand saturation**.
- Persistent expectation of new products fueled **capital over-accumulation**.
- Unfulfilled belief led to sudden correction and **crash**, worsened by process innovation.

Model:

- Incorporates process and product innovation.
- Shows the mechanism can generate a 1929-sized crash.

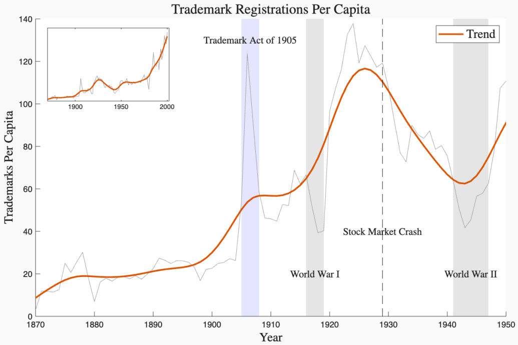
References

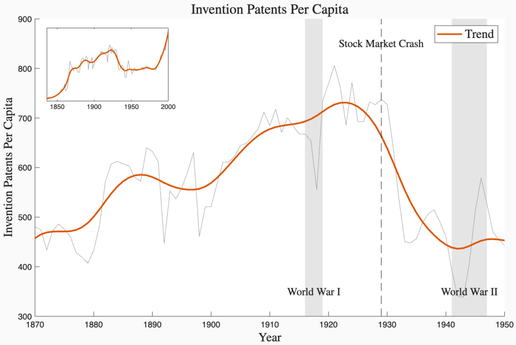
Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, 101(3):471–475.

Yorukoglu, M. (2000). Product vs. process innovations and economic fluctuations. In *Carnegie-Rochester Conference Series on Public Policy*, volume 52, pages 137–163. Elsevier.

Zeira, J. (1999). Informational overshooting, booms, and crashes. *Journal of Monetary Economics*, 43(1):237–257.

More on Product Innovation (1/2)

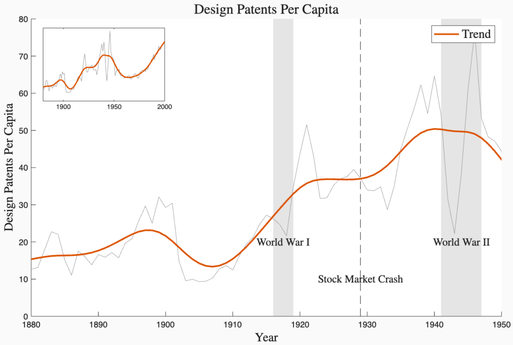


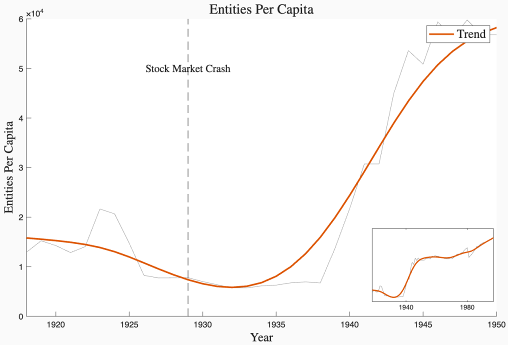


Design Patents (*left*) and Entities (*right*). Source: *Historical Statistics* (2006).

More on Product Innovation (2/2)

Back

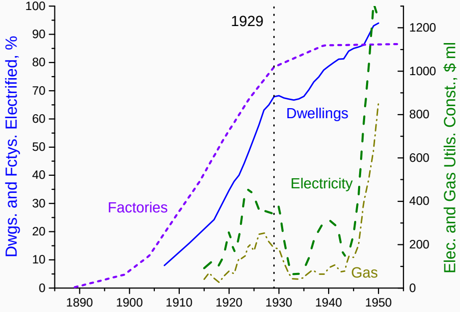


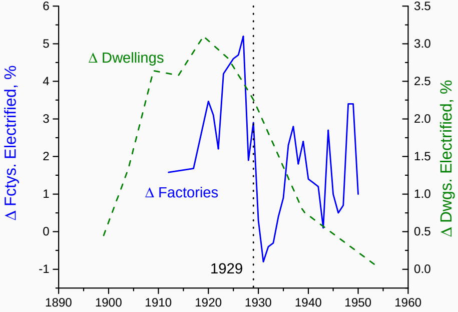


Design Patents (*left*) and Entities (*right*). Source: *Historical Statistics* (2006).

More on Satiation - Electrification

Back

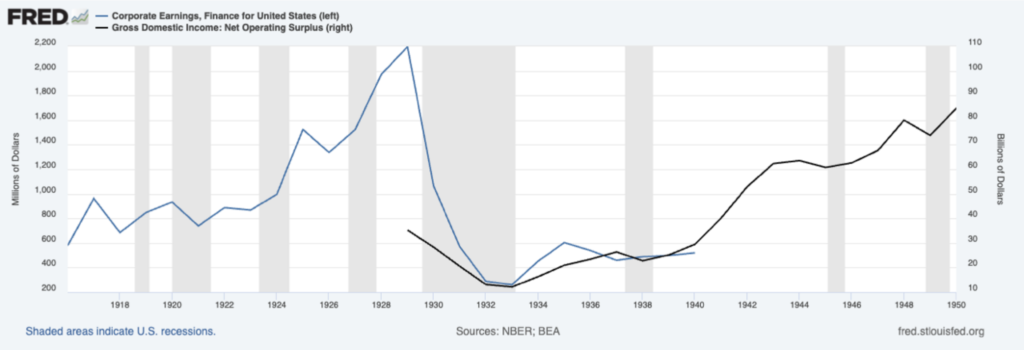




Evidence - Profits

Back

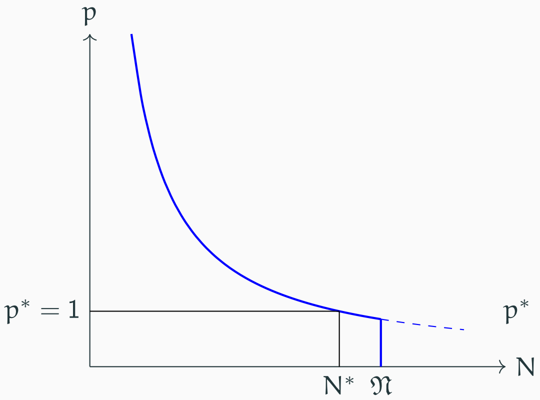
- Profits took off in the latter part of the 1920s.

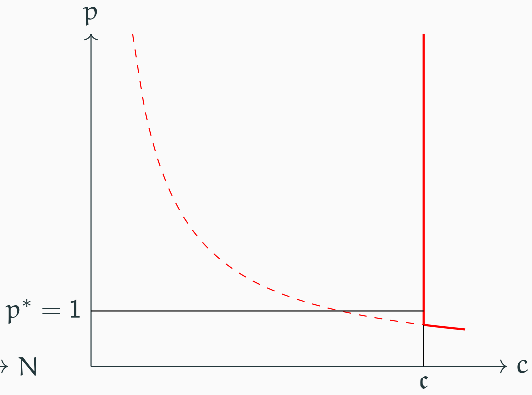


Evolution of Profit proxies. Source: FRED.

Model - Graphical Representation of Zone 1

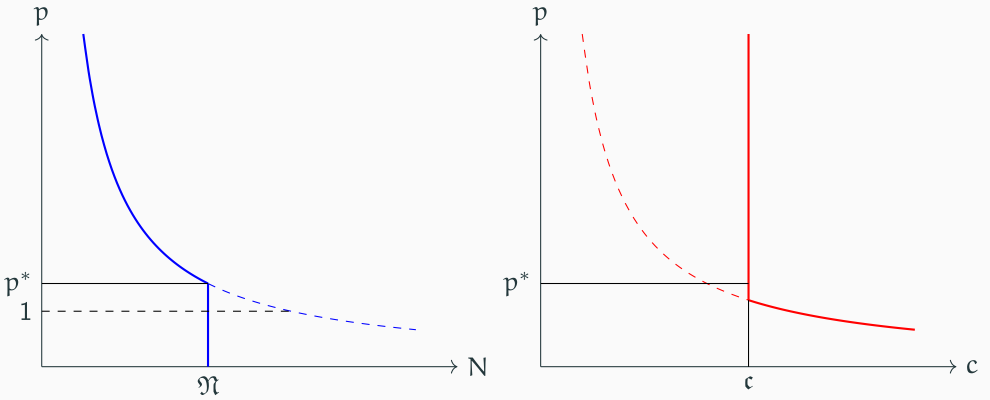
Back





Model - Graphical Representation of Zone 2

Back

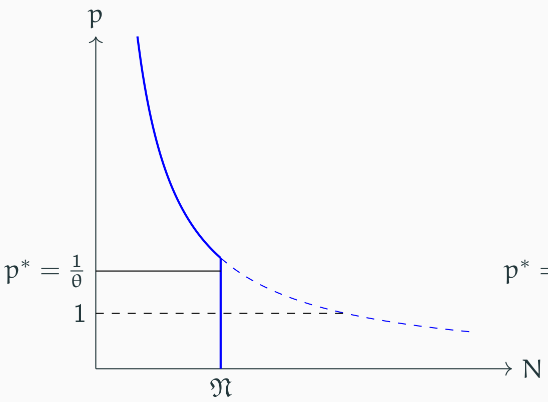


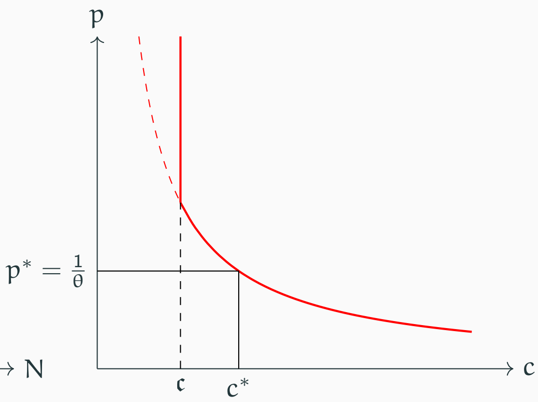
Model - Graphical Representation of Zone 3

Back

When does the economy switch to zone 3?

- If at $p = \frac{1}{\theta}$, household maximization implies $N^* = \mathfrak{N}$ and $c^* > \mathfrak{c}$.





Model - Labor Conditions

Back

- **Zone 1 and 2**:

$$\left[\frac{\partial}{\partial l_t}\right]: \quad \frac{\alpha}{\theta}\frac{1}{N_t \mathfrak{c}}\frac{w_t}{p_t} = (1-\alpha) l_t^{\chi}$$

  - Since $p_t^{Z2} > p_t^{Z1} = 1$, labor supply will be lower in zone 2 than in zone 1, *ceteris paribus*.

- **Zone 3**:

$$\left[\frac{\partial}{\partial l_t}\right]: \quad \alpha\frac{1}{\mathfrak{N}_t c_t}\frac{w_t}{1/\theta} = (1-\alpha) l_t^{\chi}$$

  - Since $\theta < 1$ (lower marginal utility of consumption), labor supply will be lower in zone 3 than in zone 1, *ceteris paribus*.
  - The relationship between zone 2 and 3 is ambiguous (at the switching point):
    - lower marginal utility of consumption in zone 3;
    - potentially higher real wage.

Intuition for Transition

Back

- Process innovation goes on throught the entire transition path.
- This makes agents **more optimistic** about innovation in general.
- Agents progressively **revise down the belief of a stall** as the economy moves up along the diagonal.

Solution Algorithm

- Solution algorithm: **nested multiple shooting**.
  - T **deterministic** stall paths $\Longrightarrow$ find $\{MB_k^{\rightarrow}\}_{t=1}^{T}$ given $\{k_{t+1}\}_{t=1}^{T}$.
  - **stochastic path** $\Longrightarrow$ find $\{k_{t+1}\}_{t=1}^{T}$ given initial condition k_0 and $\{MB_k^{\rightarrow}\}_{t=1}^{T}$.
- Terminal conditions:
  - Rebound path converges to a zone 1 BGP ($g_{\mathfrak{N}} \geqslant g_z$).
  - Stall paths converge to a zone 3 BGP ($0 = g_{\mathfrak{N}} < g_z$)

Solution Algorithm - Detailed Description

Back

Rewrite the Euler equation as

$$MC_t^{\uparrow}\left(k_t^{\uparrow}, k_{t+1}^{\uparrow}\right) = \beta\left(1-\phi_{t+1}^{t+1}\right) MB_{t+1}^{\uparrow}\left(k_{t+1}^{\uparrow}, k_{t+2}^{\uparrow}\right) + \beta\phi_{t+1}^{t+1} MB_{t+1}^{\rightarrow}\left(k_{t+1}^{\uparrow}, K_{t+1}^{\rightarrow}\left(k_{t+1}^{\uparrow}\right)\right), \tag{1}$$

a second-order difference equation in $k_t^{\uparrow}$, $k_{t+1}^{\uparrow}$, and $k_{t+2}^{\uparrow}$.

Algorithm:

0. Let the initial condition be $\bar{k}_0$ and the terminal ones be $\bar{k}_T^{\uparrow} = k_T^{BGP1}$ and $\bar{k}_T^{\rightarrow} = k_T^{BGP3}$.
1. Guess value $k_1^{\uparrow}$ and:
   1.1 solve for $\{k_{\tau+2}^{\rightarrow}\}_{\tau=t}^{T-2}$ through multiple shooting iterating forward the second-order difference equation

   $$MC_{t+1}^{\rightarrow}\left(k_{t+1}^{\rightarrow}, k_{t+2}^{\rightarrow}\right) = \beta MB_{t+2}^{\rightarrow}\left(k_{t+2}^{\rightarrow}, k_{t+3}^{\rightarrow}\right),$$

   given initial condition $k_1^{\uparrow}$ and terminal condition k_T^{BGP3}.

   1.2 solve for $\left\{k_{t+2}^{\uparrow}\right\}_{t+0}^{T-2}$, using (1) and $K_{t+1}^{\rightarrow}\left(k_{t+1}^{\uparrow}\right)$.
2. Check for convergence $(k_T^{\uparrow} - k_T^{BGP1} < \epsilon)$, otherwise update guess $k_1^{\uparrow}$ and restart from step 1.

Simulation - Parameter Values

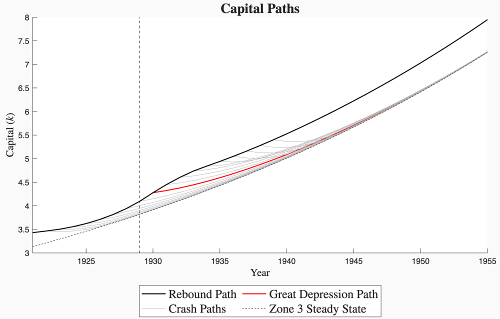
Back

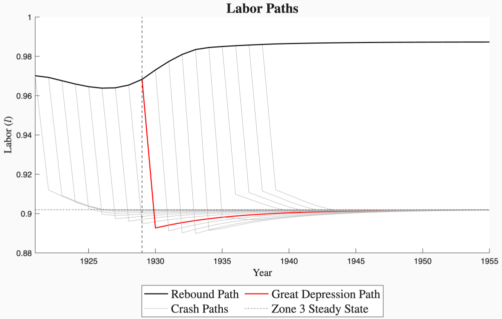
Table 1: Parameter Values

Parameter	**Value**	**Basis**
	Preferences	
α	0.5	$\approx$ 75% consumption share
θ	0.9	$\approx$ 11% mark-up rate
χ	1.33	Chetty et al. [2011]
β	0.96	Standard
	Technology	
γ	1/3	Standard
δ	0.08	Standard
	Technological Progress	
g_z	1.025	GDP growth rate 1921–1929
$g_{\mathfrak{M}}^{1921-1929}$	1.01	
	Adjustment Costs	
ϕ	2×10^{-6}	GDP decline 1929–1930
κ	5×10^{-3}	
ω	0.9	

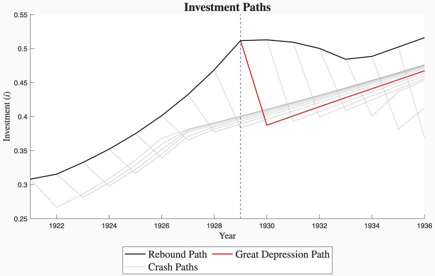
Simulation - Full Rebound and Stall Paths

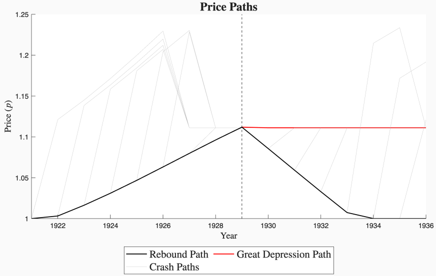
Back

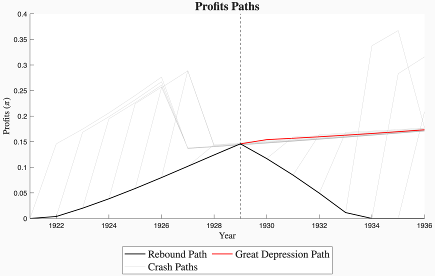




Simulation - Other Endogenous Variables

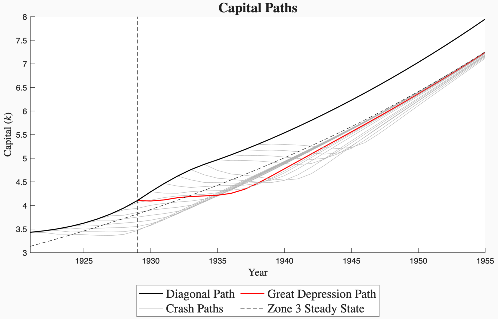


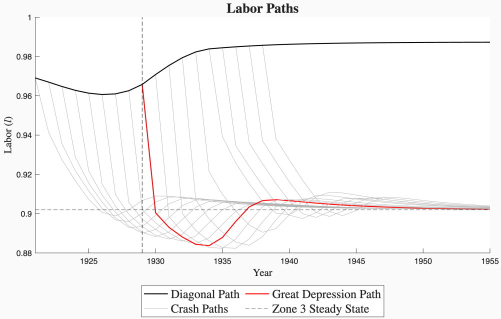




Simulation - Full Rebound and Stall Paths with Adjustment Costs

Back





Simulation - Other Endogenous Variables with Adjustment Costs

Back