Population Aging, Skill Composition, and Technology Adoption

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Introduction

- The workforce is aging in most developed economies
  - Share 45+ in US: $\uparrow$ from 38.5% in 2004 to 42%, and to 44.5% by 2034 [BLS]
- Much of macro literature studies aging through Labor Force growth
  - The role of age composition has has received far less attention
- Aging affects the human capital of the workforce
  - Older workers bring more experience accumulated on the job. [Ben-Porath, 1967]
  - Younger workers bring more up-to-date skills [Deming and Noray, 2020]

Introduction

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  - Share 45+ in US: $\uparrow$ from 38.5% in 2004 to 42%, and to 44.5% by 2034. [BLS]
- Much of macro literature studies aging through LF growth
  - The role of age composition has has received far less attention.
- Aging affects the human capital of the workforce ...
- ... with ambiguous effects on aggregate labor productivity
  - Old (%) $\uparrow$ $\implies$ experience $\uparrow$ $\implies$ Labor Productivity $\uparrow$
  - Young (%) $\downarrow$ $\implies$ tech-skills $\downarrow$ $\implies$ Technology Adoption $\downarrow$ $\implies$ Labor Prod. $\downarrow$

Q. What's the impact of aging on productivity via Skill Composition and Tech Adoption?

This Paper

Theoretical Framework

- Two technologies and two skills: Frontier/Standard, Tech-Skills/Experience
- Theory decomposes the productivity effect of aging into
  - Direct Skill-Composition vs Technology Adoption
- Identification: Responses of firms's outcomes to adoption events

Empirics

- Event studies around technology adoption using German Data
  - Support model's predictions + provide estimation targets.

Quantitative Results (Preliminary)

- Projected 2050s aging: labor productivity +0.5%.
- Net effects masks opposing forces: Experience↑, Tech Adoption↓
- Training subsidies outperform adoption subsidies for productivity gains/dollar

Related Literature

- Effect of population aging on firm dynamics and TFP.

  Maestas et al. [2023], Aksoy et al. [2019], Hopenhayn et al. [2022], Bloom et al. [2020], Karahan et al. [2024], Angelini [2023], Inokuma and Sanchez [2023], Peters and Walsh [2021]

  - Our Paper: Focus on Age Composition + Trade-off for Skill Composition

- Vintage human capital and technology diffusion.

  Chari and Hopenhayn [1991],MacDonald and Weisbach [2004],Adão et al. [2024], Violante [2002]

  - Our Paper: Skills that are portable across tech. + impact of population aging.

- Workers Age and New Technologies

  Lipowski [2024],Deng et al. [2024],Deming and Noray [2020], Acemoglu and Restrepo [2022]

  - Our Paper: Young workers are an important input for technology adoption of firms.

Outline

1. Theoretical Framework
2. Quantitative Model
3. Empirical Analysis
4. Quantitative Analysis

Theoretical Framework

Goals

1. Show the central mechanism using the minimum set of ingredients
2. Sharp predictions: Age Composition $\Longleftrightarrow$ Tech Adoption, Labor Productivity
3. Full Model: Relax assumptions but preserve the mechanism.

Environment

- Static Model
- Two available technologies: Frontier (F) and Standard (S)
  - Require two inputs: Tech-Skills (T) and Experience (E)
- **Workers**: Measure 1, exogenous skill endowment
  - Young: Tech-skills, Low Experience $\gamma_y = (\Gamma_T, 1)$, $\Gamma_T \geq 1$
  - Old: Low Tech-skills, Experience, $\gamma_o = (1, \Gamma_E)$, $\Gamma_E > 1$
- **Homogeneous Firms**: Measure M
  - Choose: Employment, Skill Composition, Technology

Production Technologies

- Two available technologies: Frontier (F) and Standard (S)
- The production function of technology τ is

$$y_\tau = \underbrace{A_\tau}_{\text{TFP}} \underbrace{\left(T^{\alpha_\tau} \; E^{1-\alpha_\tau}\right)^\eta}_{\text{Labor Composite}}, \quad \eta < 1, \; \alpha_\tau \in [0,1].$$

- **A1. Frontier (F) has higher TFP** $\implies A_F > A_S = 1$
- **A2. Frontier (F) is more tech intensive** $\implies \alpha_F > \alpha_S$

Production Technologies

- Key Choices:
  - l: Total employment x: Share of young workers τ: Technology

- The production function of technology τ is

$$y_\tau(l,x) = \underbrace{A_\tau}_{\text{TFP}} \underbrace{l^\eta}_{\text{Size}} \underbrace{\left(\bar{T}(\underset{+}{x})^{\alpha_\tau} \ \bar{E}(\underset{-}{x})^{1-\alpha_\tau}\right)^\eta}_{\text{Skill-Mix per worker}}$$

$$\bar{T}(\underset{+}{x}) = 1 + \underbrace{(\Gamma_T - 1)x}_{\text{Gain from Young}} \qquad \text{(Tech-Skill)}$$

$$\bar{E}(\underset{-}{x}) = \Gamma_E - \underbrace{(\Gamma_E - 1)x}_{\text{Loss from Young}} \qquad \text{(Experience)}$$

Production Technologies

- Key Choices:
  - l: Total employment x: Share of young workers τ: Technology
- The production function of technology τ is

$$y_\tau(l,x) = \underbrace{A_\tau}_{\text{TFP}} \underbrace{l^\eta}_{\text{Size}} \underbrace{\left(\bar{T}(\underset{+}{x})^{\alpha_\tau}\ \bar{E}(\underset{-}{x})^{1-\alpha_\tau}\right)^\eta}_{\text{Skill-Mix per worker}}$$

- Technology: Adopting the Frontier (F) is costly and risky.
  - Invest share of output r $\Longrightarrow$ adoption rate $a = \phi(\underset{+}{r})$

Technology Adoption

- Equilibrium prices: Tech-Skills: p_T, Experience: p_E.
  - Relative tech-cost: $m = \frac{p_T}{p_E}$
- Define Technology adoption a as the share of frontier firms.

Prop. 1

Tech. adoption depends only, and negatively, on the relative tech-cost: $\frac{da(m)}{dm} < 0$.

The Central Mechanism

Thought Exp. Increase the share of old (o) at constant Labor Force.

- Shock to the skill composition: $E\uparrow$, $T\downarrow$

Prop. 2

Aging increases relative tech-price: $\frac{dm}{do} > 0$.

Prop. 3

Aging reduces technology adoption: $\frac{da(m)}{do} < 0 :$ $\underbrace{\frac{\partial a}{\partial m}}_{\textit{Frontier Premium},\ <0} \times \underbrace{\frac{dm}{do}}_{\textit{Tech Premium},\ >0}$

Prop. 4

Aging affects productivity ambiguously

$$\frac{d\mathcal{Y}}{do} = \underbrace{w_o - w_y}_{\textit{Direct Channel},\ >0} + \underbrace{(1-\eta)\times M \times \Delta y_\tau^* \times \frac{da}{do}}_{\textit{Adoption Channel},\ <0}$$

Outline

1. Theoretical Framework
2. Quantitative Model
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4. Quantitative Analysis

Quantitative Model

- Purpose of the Model
  - Central Mechanism + Quantitatively Relevant Features
  - Quantification: Impact of age composition on aggregate labor productivity
- Infinite Horizon model in continuous time
  - **Firms**: Forward Looking hiring and Technology Adoption decisions
  - **Workers**: Skill Transitions over life cycle
    - Experience: Accumulate; Tech Skills: Obsolescence + Training (endogenous)
- Search Frictions + multi-worker firms (*Pijoan-Mas and Roldan-Blanco [2024]*)
  - Frictional lab. mkt. needed to match key data targets

Agents

Workers

State variables:

- Tech skills, H/L: $\to \{\Gamma_T, 1\}$, $\Gamma_T \geq 1$
- Experience, e/i: $\to \{\Gamma_E, 1\}$, $\Gamma_E \geq 1$

$\implies$ Four types: $\mathcal{I} \equiv \{Li, Le, Hi, He\}$

Directed Search from Unemployment:

- Hiring market (W)

Firms

State variables: $(\vec{n},\ \tau,\ z)$

- Workforce Composition
- Technology: Frontier, Standard
- Idiosyncratic Productivity (Exogenous)

Choose:

- Technology (τ)
- Hiring market and effort (v, W)
- Employees' long-term contract
  - Wage (w), firing (δ), training (λ^{tr})

Optimal Contract

Production Technologies

- Two available technologies: Frontier (F) and Standard (S)

- The production function of technology τ is

$$y_\tau = \underbrace{zA_\tau}_{\text{TFP}} \underbrace{\left(T^{\alpha_\tau} E^{1-\alpha_\tau}\right)^\eta}_{\text{Labor Composite}}, \quad \eta < 1, \; A_0 > A_1 = 1, \; \alpha_0 > \alpha_1$$

$$s.t. \qquad T = \Gamma_T \underbrace{(n_{Hi} + n_{He})}_{\text{High}} + \underbrace{(n_{Li} + n_{Le})}_{\text{Low}} \qquad \text{(Tech-Skill)}$$

$$E = \Gamma_E \underbrace{(n_{He} + n_{Le})}_{\text{High}} + \underbrace{(n_{Li} + n_{Hi})}_{\text{Low}} \qquad \text{(Experience)}$$

Production Technologies

- At any instant, two technologies are available:
  - Frontier (F) and Standard (S)

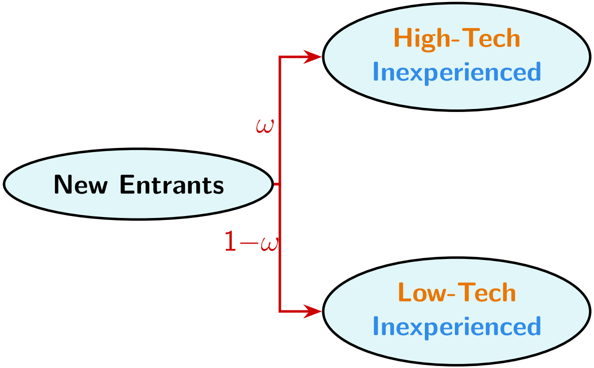
- The production function of technology τ is

$$y_\tau = \underbrace{zA_\tau}_{\text{TFP}} \underbrace{\left(T^{\alpha_\tau} \; E^{1-\alpha_\tau}\right)^\eta}_{\text{Labor Composite}}, \quad \eta < 1$$

- **Technology Adoption.** Firms can invest $r \geq \mathbf{0}$ to affect the following
  - **Tech-Upgrade.** Standard become Frontier at rate $q_{10}(\underset{+}{\mathbf{r}})$
  - **Tech-Downgrade.** Frontier become Standard at rate $q_{01}(\underset{-}{\mathbf{r}})$
  - **Obsolescence.** Standard firms exit the market at rate $q_{12}(\underset{-}{\mathbf{r}})$

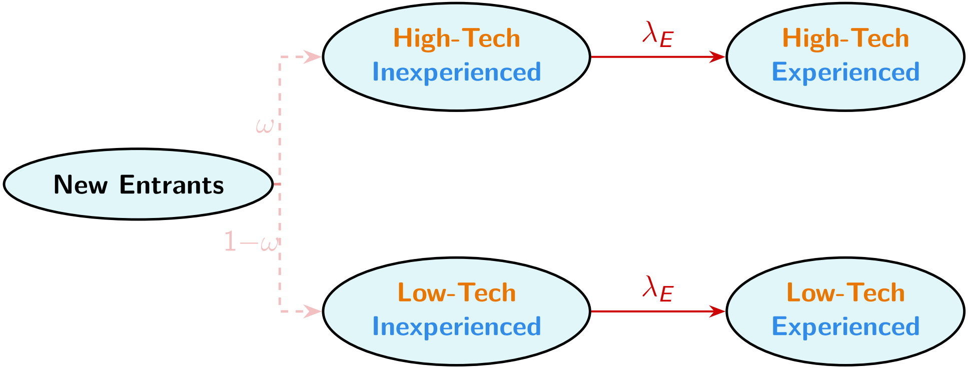
Tech Transitions Cyclical Upgrade

Workers: Skill Transitions



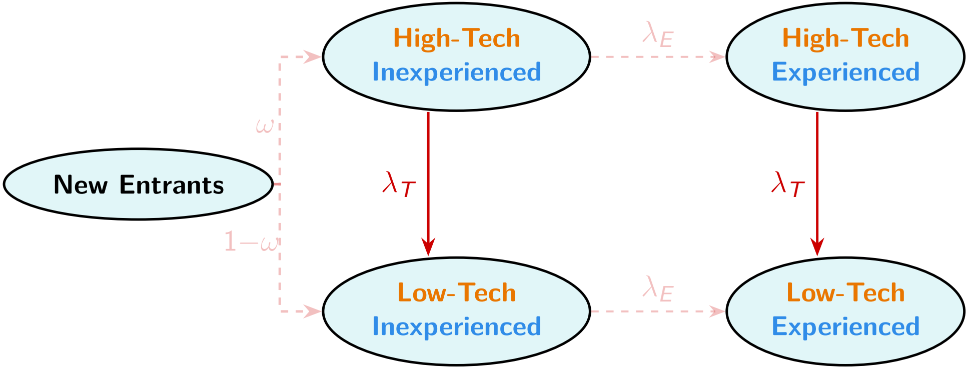
- A share ω of new workers start with **High-Tech**, all have **Low-Experience**

Workers: Skill Transitions



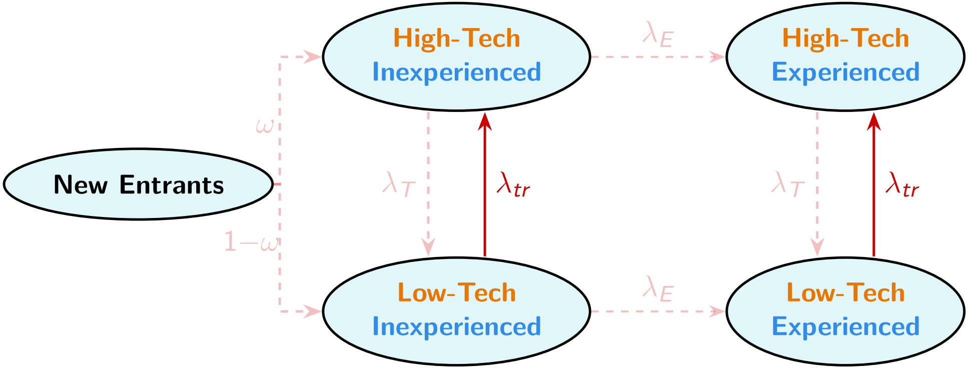
- Workers accumulate experience at rate λ_E.

Workers: Skill Transitions



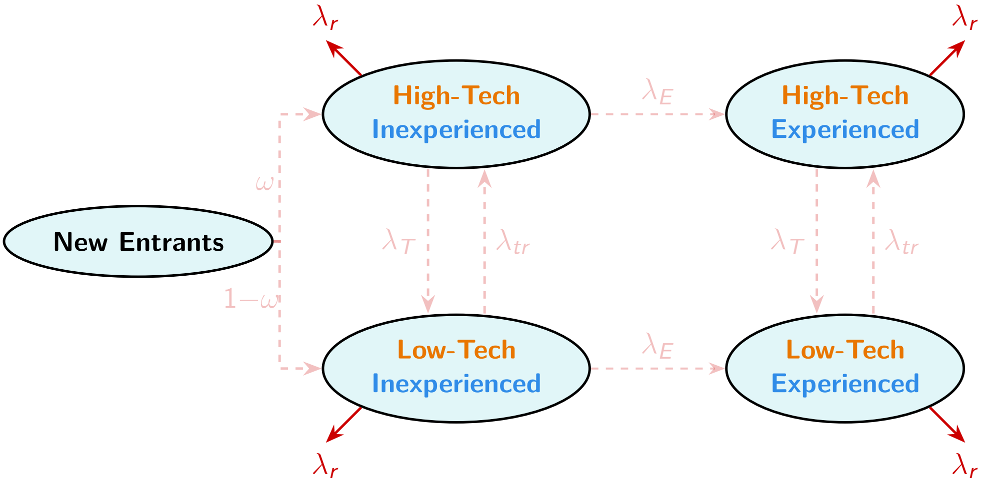
- Tech skills become obsolete at rate λ_T.

Workers: Skill Transitions



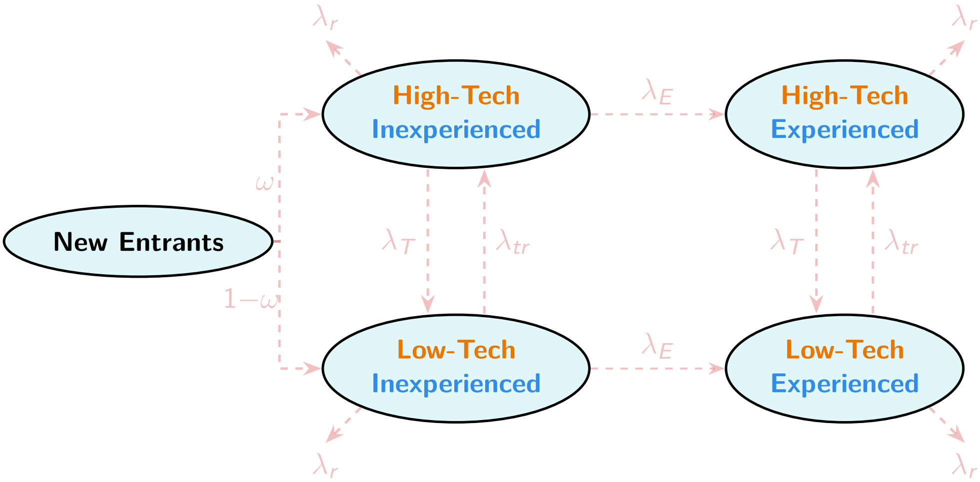
- Firms can retrain workers at rate λ_{tr}.

Workers: Skill Transitions



- Workers retire at rate λ_r.

Workers: Skill Transitions



- Law of Motion of $\mathcal{L}$: $\underbrace{\mu_t}_{\text{New Entrants}} = \mathcal{L}_t \underbrace{\mathbf{g}}_{\text{LF growth rate}} + \underbrace{\lambda_r \mathcal{L}_t}_{\text{Retirement}}$

Closing the model

- Free-Entry Condition $\iff$ New Entrant Firms
  - Flow cost κ of opening a business
  - Draw productivity $z \sim F_z$
  - Probability p_τ of being Frontier (exogenous)
- We solve for a stationary equilibrium per-capita

Timeline Equilibrium Entrant Problem

Model to Data: Intuition

Q. How can we quantify the impact of age composition on productivity?

STEP 1. Facts supporting the assumptions: $\alpha_0 > \alpha_1$, $\Gamma_T > 1$.

STEP 2. Identify key parameters: A, α, Γ

- Technology:
$$y_\tau(l, x) = \underbrace{A_\tau}_{\text{TFP}} \underbrace{l^\eta}_{\text{Size}} \underbrace{\left(\bar{T}(\underset{+}{x})^{\alpha_\tau} \; \bar{E}(\underset{-}{x})^{1-\alpha_\tau} \right)^\eta}_{\text{Skill-Mix per worker}}$$

- Theory Predictions: Firm responses after technology adoption ($S \to F$)
  1. Employment Response: Δl increasing in A
  2. Age Composition: Δx increasing in $\Delta\alpha$
  3. Wage Response: $\frac{\Delta w_y}{\Delta w_o}$ increasing in Γ_T

- Strategy: Event Study analysis around firms Tech Adoption

Outline

Data

Main Goal: Link firm-level tech adoption to info on workforce composition.

German Admin Data: Matched Employer Employee + Tech Adoption Surveys.

- Linked Employer-Employee (1993-2021)
  - Firms: Workforce Composition, On-the-job training.
  - Workers: Employment Histories, Wage, Education, Demographics.
- LPP (2015, 2019, 2021): Survey of ICT adoption
  - Digital: e.g., IoT, Big Data Analytics, AI, Virtual Reality.
  - Automation: e.g., Robots, Drones, Additive Manufacturing.

Tech Adoption: At least one technology adopted in the last two years. LPP Details

Fact 1 - Young Workforce Predicts Higher Adoption

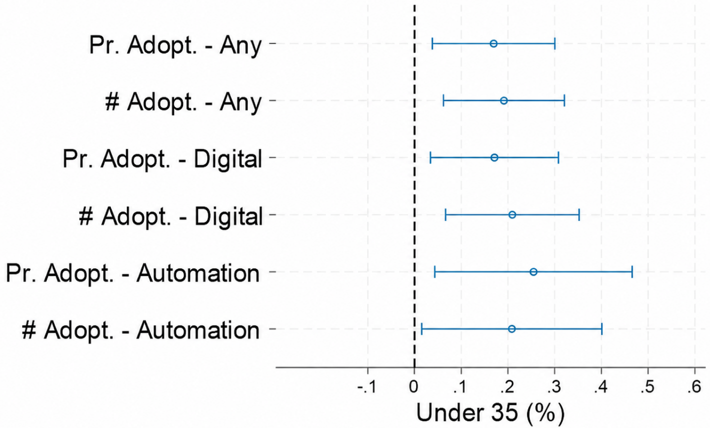
Empirical Specification

$$\mathbb{I}(\text{Adopt})_t = \alpha + \beta \text{ Under 35 } (\%)_{t-1} + \Gamma X_{t-1} + \varepsilon_t$$

- Baseline Controls (X_{t-1}): Firm Size, Firm age, Avg. Wage, College (%), Hiring Rate, Type of Owner.
- Fixed Effects: Sector 3-digit, State, Year, # Tech already used.

Fact 1 - Young Workforce Predicts Higher Adoption

- Moving ↑ by 1-SD of distribution of U35 % $\implies$ Pr. Adopt ↑ by 20%
  $\implies$ # Adoption ↑ by 20%



Under 40 Full controls

Fact 2 - Young Workers benefit from Technology Adoption

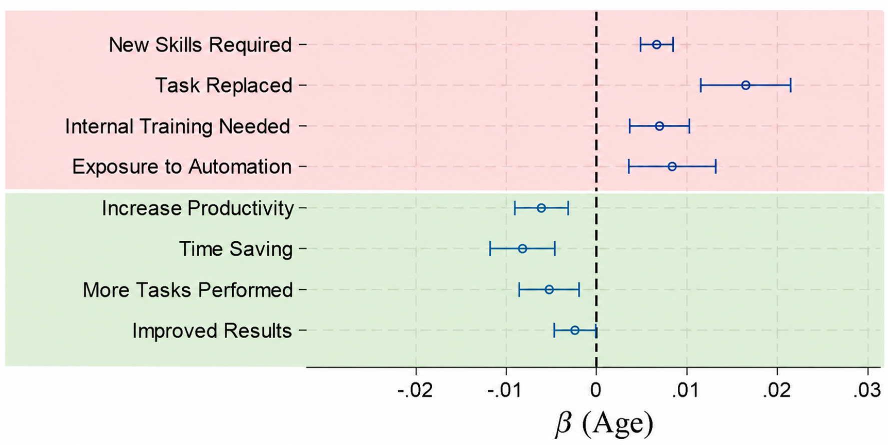
Empirical Specification

$$\mathbb{I}(\mathsf{Y}_{it}) = \alpha + \beta \ \mathsf{Age}_{it} + \Gamma X_{it} + \underbrace{\gamma_{f(it)}}_{\text{Firm FE}} + \varepsilon_{it}$$

- Outcomes (Y_{it})
  - Survey Questions on Impact of Tech-Adoption
- Baseline Controls (X_{it}): Wage, Occupation Tenure, Firm Age.
- Fixed Effects: Firm, Year, Task Complexity, College, Occupation (3-digit)

Fact 2 - Young Workers benefit from Technology Adoption

- Young workers report productivity gains from technology adoption



Full Controls Task Questions

Model-Data Mapping - Event Study Around Adoption

Empirical Strategy

- Compare dynamic outcomes of adopting vs non-adopting establishments.
- Matching-bins: Sector, Log Employment (t-1), Age Composition (t-1).

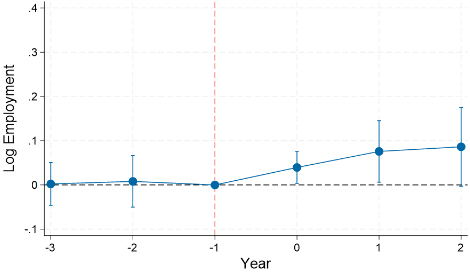
Empirical Specification

$$y_{ft} = \alpha_f + \gamma_t + \sum_k \underbrace{\theta_k D_{ft}^k}_{\text{Non-Adopters}} + \sum_k \underbrace{\beta_k (D_{ft}^k \times \text{Tech Adoption}_f)}_{\text{Adopters}} + \varepsilon_{ft},$$

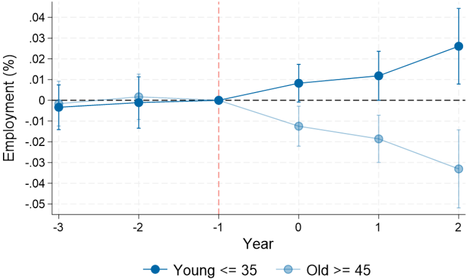
- Outcomes (y_{ft}): Log-Employment (Δl), Age Composition (Δx), Wages ($\frac{\Delta w_y}{\Delta w_o}$)
- Fixed Effects: Establishment, State, Year, Age bins.

Model-Data Mapping - Event Study Around Adoption

- Firms expand employment disproportionately by hiring young.



(a) Log Employment

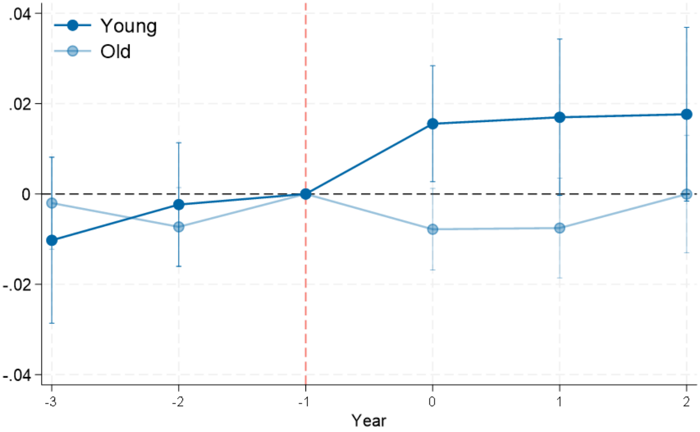


(b) Age Composition

Log Empl. Under 40 College Automation Tech General Empl. Shocks

Model-Data Mapping - Event Study Around Adoption

- Young Incumbents enjoy wage gains after adoption.



Empirics - Summary

- **Facts** $\implies$ Age is correlated with Tech-Skills.

  1. Younger workforce predict higher adoption
  2. Young workers benefit from adoption

- **Event Study** $\implies$ Targets in estimation

  1. Firms expand employment... $\Delta l \iff A$
  2. ... disproportionally hiring young workers $\Delta x \iff \Delta\alpha$
  3. Young incumbents receive wage gains Γ_T

Outline

1. Theoretical Framework
2. Quantitative Model
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Calibration

- Calibrate the steady state to Germany (2005-2019)
- Crucial Parameters
  - A, α_0, Γ^T: Event Study Results (employment, age comp., wages)
  - $\Gamma^E, \omega, \lambda_T, \lambda_E$: Dispersion and wage growth in early career
  - Training Cost: Share of employees participating in OJT
- Other parameters
  - Tech Transitions: Adoption Rate+Exit Rate by tech status
  - Labor Mkt + Firm Dynamics parameters: standard moment

Internal Calibration External Parameters

Calibration

Technology and Skills

Parameter	Value	Target	Data	Model
A_0	1.120	Firm log-employment DID	0.100	0.102
$\frac{\alpha_0}{\alpha_1}$	1.432	Firm young-employment-share DID	0.030	0.0286
Γ_T	1.804	Young-minus-non-young log-wage DID	0.020	0.017
Γ_E	2.121	Average log-wage IQR over the first 5 years	0.61	0.57
λ_T	0.0339	10-year change in the log-wage IQR	−0.040	−0.0455
λ_E	0.0241	5-year change in the P50–P25 log-wage gap	−0.030	−0.0343
ω	0.654	Entrant share below the stationary P30 wage cutoff	0.350	0.327
κ_e	54.213	Six-month on-the-job training rate	0.050	0.0510
ψ_{01}	8.510	Frontier-to-laggard transition rate	0.130	0.128
ψ_{12}	22.136	Laggard exit rate	0.056	0.0558
χ_{10}	1.025	Share of active high-tech firms	0.300	0.295

Labor Market and Firms (standard)

Parameter	Value	Target	Data	Model
κ_v	1.743	Unemployment rate	0.0834	0.0995
κ_δ	15.000	Dismissal rate	0.0152	0.0251
s^W	0.0050	Monthly employment-to-unemployment rate	0.0052	0.0074
s^F	0.00597	Annual firm entry rate	0.0506	0.0515
κ	4,369.554	Average log firm size	1.850	1.855
b	0.300	Unemployment income relative to output per worker	0.700	0.54

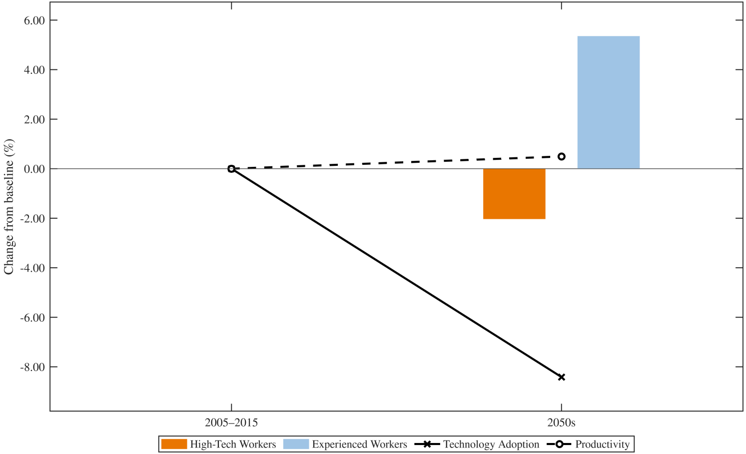
Notes:

Thought Experiment

- Calibrate the steady state of the economy to Germany 2005-2019
- Solve two counterfactual economies by targeting the annual growth rate of LF
  1. 2005-2015: $g \approx 0.31\%$
  2. 2050s (projected): $g \approx -0.24\%$

Skills and Technology Adoption

- Aging changes the skill mix: Experience$\uparrow$, Tech Skills $\downarrow$
- Equilibrium Response: Technology Adoption$\downarrow$, Productivity $\left(\frac{Y}{E}\right) \approx$



Productivity Decomposition

- Calibration predicts a modest effect of aging on productivity.

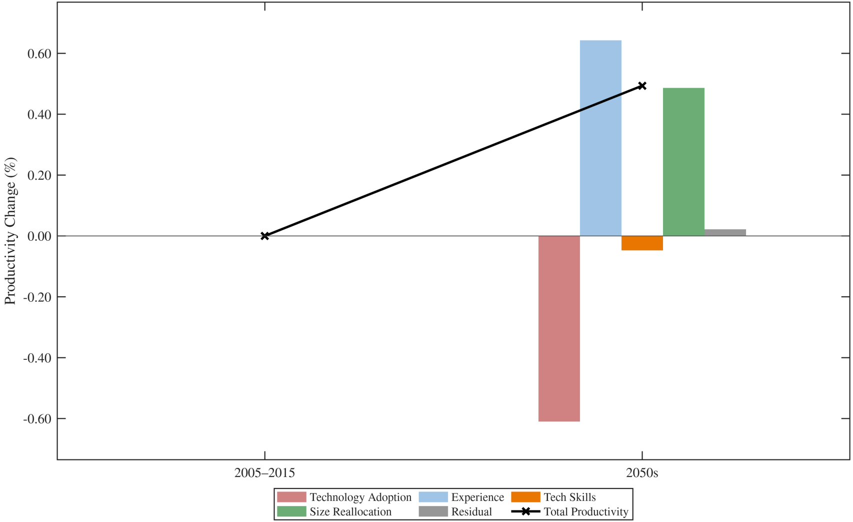
- **However**, overall result masks offsetting impact of opposite forces.

  1. Skill composition channel $\implies$ experienced workers $\uparrow$, tech workers $\downarrow$
  2. Technology adoption channel $\implies$ tech adoption $\downarrow$
  3. Size reallocation channel $\implies$ firm distribution shifts toward incumbents

Derivations

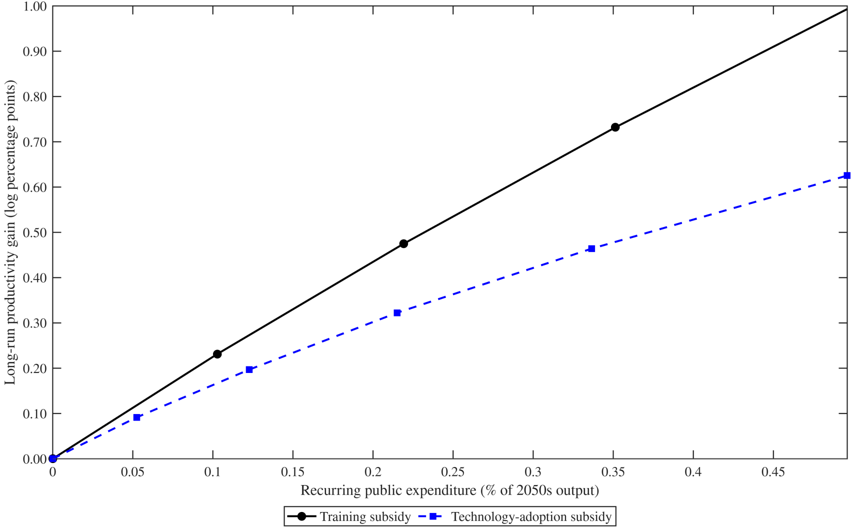
Productivity Decomposition

- Two opposite forces: Technology Adoption (-0.63 %), Experience (+0.62%).



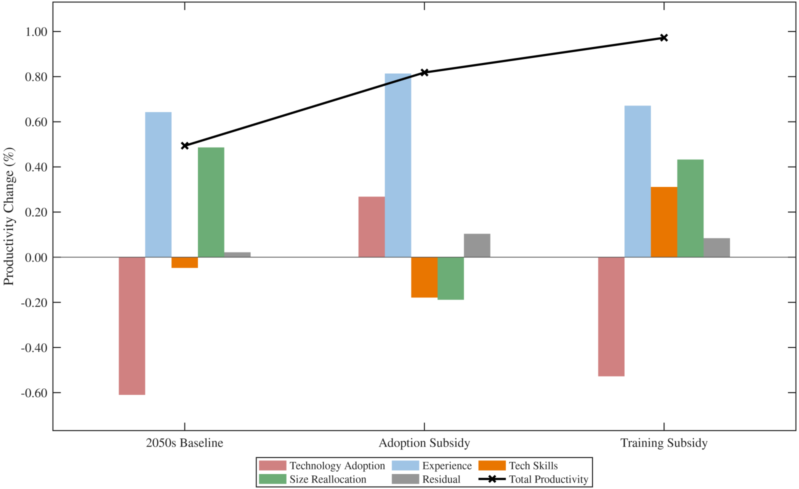
Subsidies: Adoption vs Training

- Training subsidies outperform adoption subsidies for productivity gains/dollar



Subsidies: Adoption vs Training

- **Adoption Subsidy** Too few Tech Workers $\Longrightarrow$ mitigate the benefits
- **Training Subsidy** $\uparrow$ Tech Workers $\Longrightarrow$ spillover on $\uparrow$ Adoption



Conclusions

- Population Aging alters the skill composition of the Labor Force
- Two forces at play push productivity in opposite directions
  - Experience$\uparrow$, Tech Adoption$\downarrow$
- Build a quantitative theory and discipline it using micro-data
- We Find: Experience and Tech Adoption nearly offset each others
- To mitigate Tech Adoption$\downarrow$: training subs. more effective than adoption subs.

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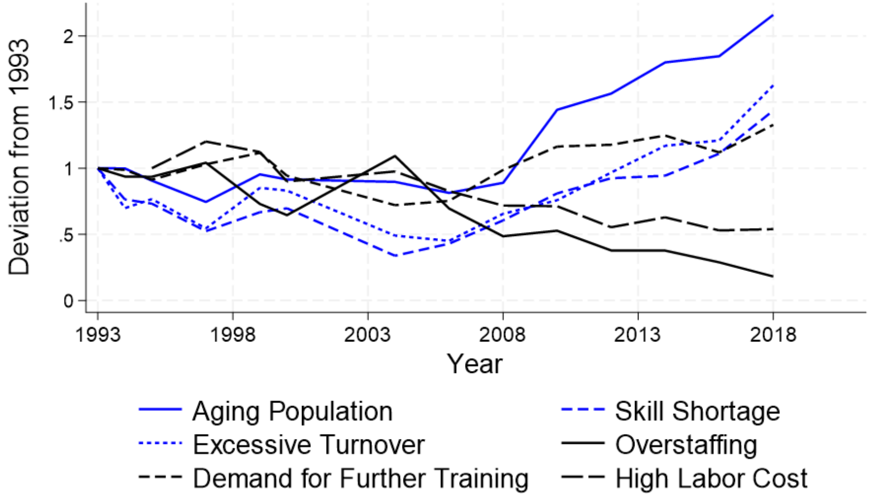
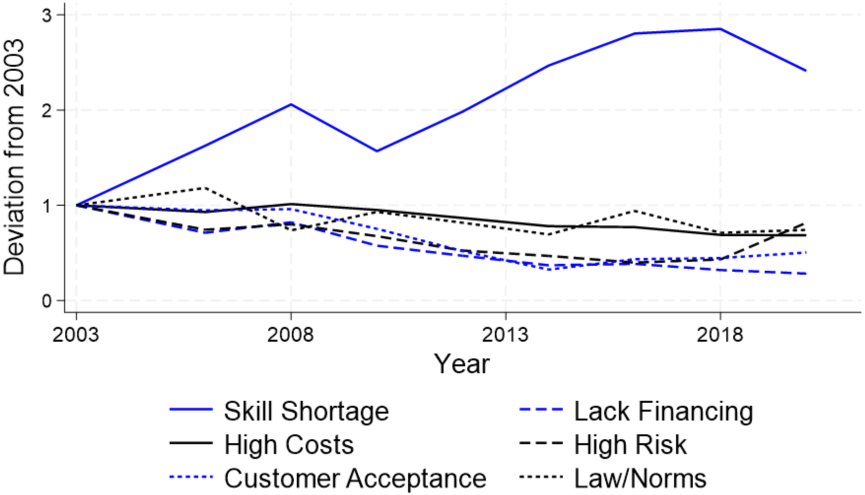
Main Concerns About Staffing



Back

Main Constraints to Innovation



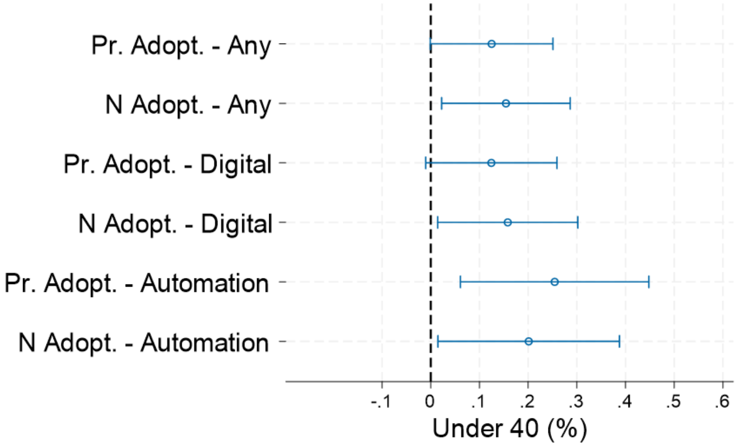
Back

LPP-ADIAB Overview

- Sample: $\approx$ 750 establishments per wave.
  - Representative of firms $\geq$ 50 employees.
  - Representative survey of employees are surveyed (7 per firm avg.)
- Advanced technologies surveyed
  - Digital: e.g., IoT, Big Data Analytics, AI, Virtual Reality.
  - Automation: e.g., Robots, Drones, Additive Manufacturing.
- Usage Options (for each tech.)
  - Use $<$ 2 years ago $\rightarrow$ Adoption Event at $t-2$.
  - Use $>$ 2 years ago $\rightarrow$ Control/Exclude.
  - Not Use.

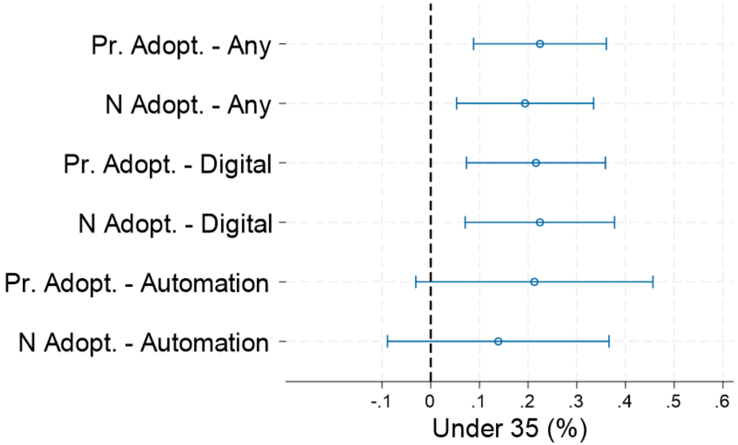
Back

Young Workforce Predicts Higher Adoption



Back

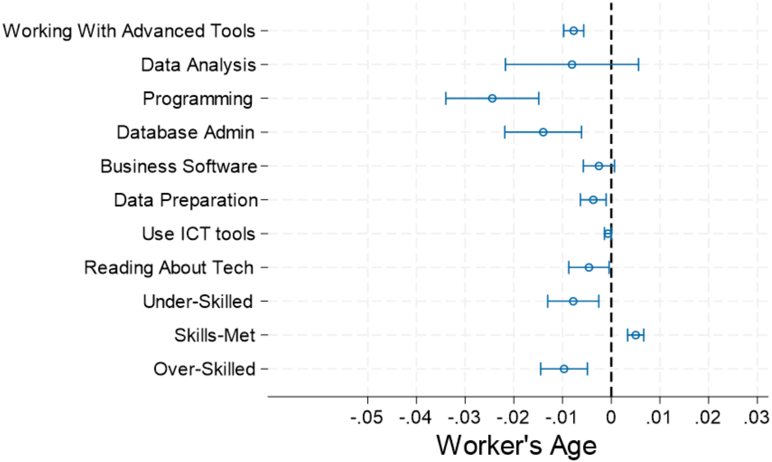
Young Workforce Predicts Higher Adoption



Back

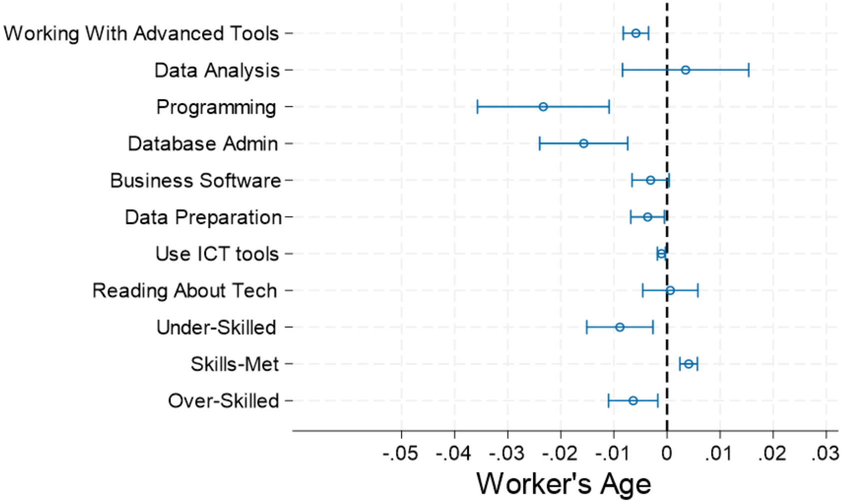
Young Perform More Tech-Related Tasks

- Coefficient: 1 year more $\Longleftrightarrow$ Pr. Answer Yes.



Back

Young Perform More Tech-Related Tasks

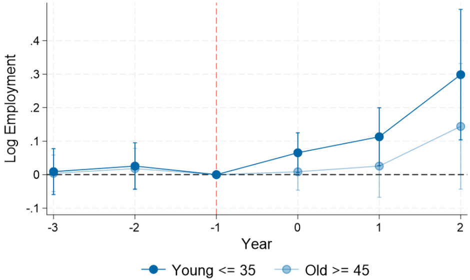


Back

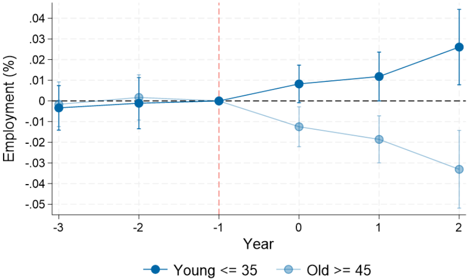
Young Report Productivity Gains for Tech Adoption

Back

Event Study - Employment Share



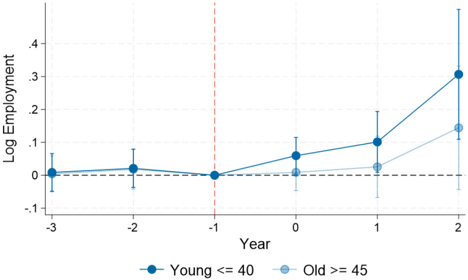
(a) Log Employment



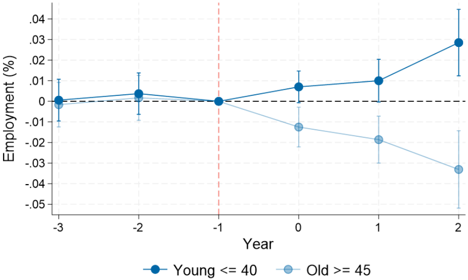
(b) Employment Share

Back

Event Study - Under 40



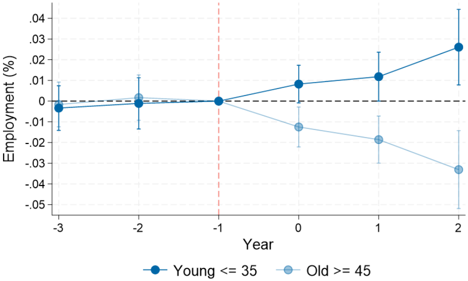
(a) Log Employment



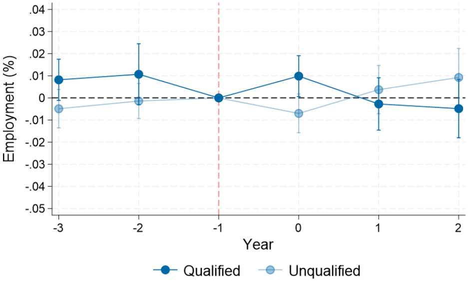
(b) Employment Share

Back

Event Study - College Heterogeneity



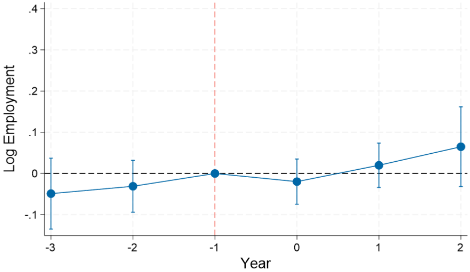
(a) Age Heterogeneity



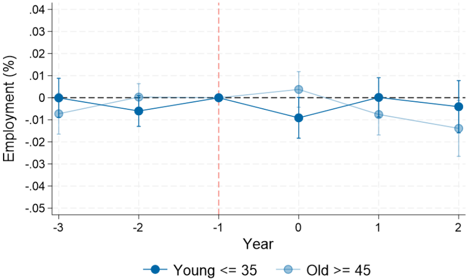
(b) College Heterogeneity

Back

Event Study - Automation Technologies



(a) Log Employment

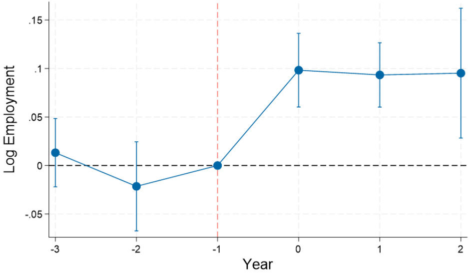


(b) Age Heterogeneity

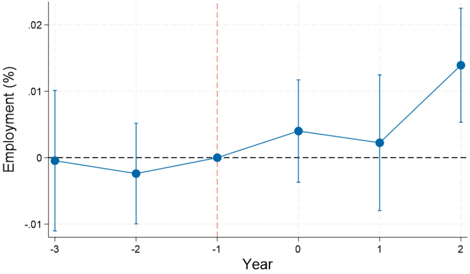
Back

Event Study - General Employment Shocks

- Sample: Future adopters before windows of adoption.
- Residualize yearly emplyment growth by sector and state trends.
- Empl. Shocks: resid. growth above 75th percentile.



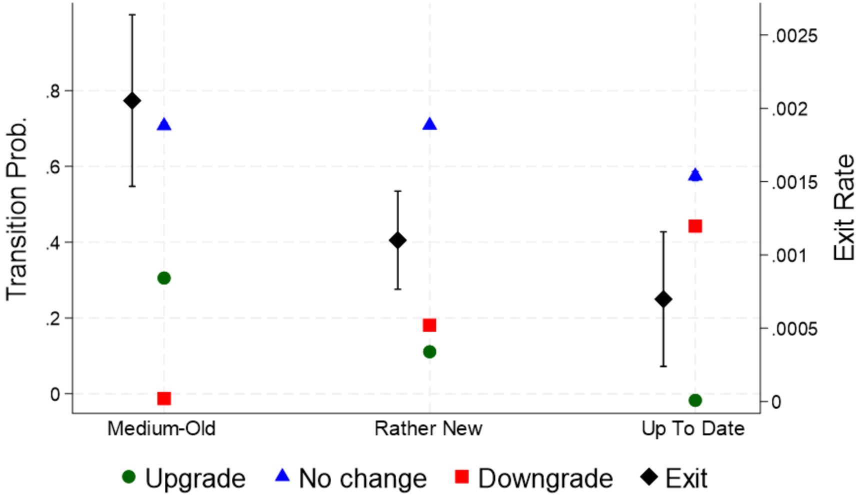
(a) Log Employment



(b) Share Under 35

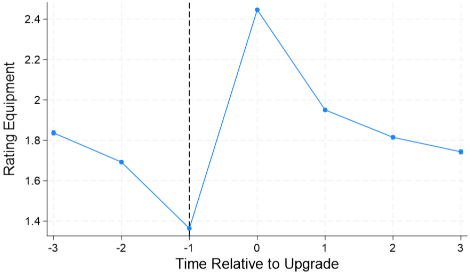
Back

Tech Transition in the Data

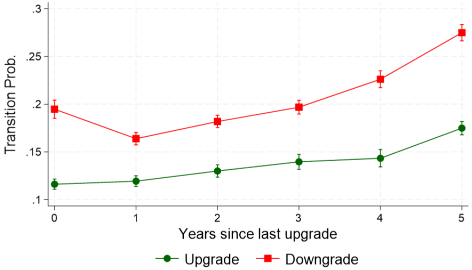


Back

Cyclicality of Tech Upgrade



(a) Equipment Rating



(b) Upgrade - Downgrade

Back

Optimal Dynamic Contract

- **Dynamic contracts**

$$\mathcal{C}(s) = \Big\{ \{w_i,\ v_i,\ \delta_i,\ W_i'(s_i^+)\}_{i\in\mathcal{I}},\ \ \lambda^{tr},\ \ q_\tau \Big\}$$

subject to promise keeping constraint + worker's participation constraint.

- Wage: $w_i,\ \forall i$
- Vacancies: $v_i,\ \forall i,$ at cost $C_v(v_i)$
- Firing Rate: $\delta_i,\ \forall i,$ at cost per worker $C_\delta(\delta_i)$
- Cont. Promise: $W_i'(s'),\ \forall i, s'$
- Training Rate: $\lambda_i^{tr},\ i \in \{Li, Le\},$ at cost per worker $C_\lambda(\lambda^{tr})$

- Restrict to: Complete Contracts + Markov Perfect Eq.

Optimal Solution Unemployed Joint Surplus

Back

Unemployed Workers

The value of an unemployed worker of type $i \in \mathcal{S}$ is

$$(\rho+\lambda^r)\mathcal{U}_i = \max_W \Big\{ b + \underbrace{\mu_i(W)}_{\text{Job Finding Rate}} \max\{W-\mathcal{U}_i, 0\} + \underbrace{\lambda_T(\mathcal{U}_{L,s_P}-\mathcal{U}_{H,s_P})}_{\text{Tech Obsolescence}} \underbrace{\mathbb{1}\{s_T=H\}}_{\text{High-tech Worker}} \Big\}$$

$$\implies \mu_i(W) = \frac{(\rho+\lambda^r)\mathcal{U}_i - b - \lambda_T(\mathcal{U}_{L,s_P}-\mathcal{U}_{H,s_P})\mathbb{1}\{s_T=H\}}{W-\mathcal{U}_i}, \qquad s_P \in \{L,H\}$$

Trade-off: job-finding rate is **inversely related** to the contract's promised value W

- $W\uparrow \implies$ submarket (i,W) attracts more workers $\implies \mu_i(W)\downarrow$, wait time $\uparrow$
- Continuum of active submarkets maximizing unemployment value.

Back

Summary Timeline Back

Beginning of period t → **Production** → **Endogenous Decisions** → **Transitions** → *New period $t+1$*

Beginning of period t

State Variables:

- Workforce $\vec{n} = \{n_s\}_s$
- Productivity z
- Technology τ

Production

- firms produce $y(z, \tau, \vec{n})$
- wages are paid

Endogenous Decisions

Firms choose:

- hiring efforts $\{v_s\}_s$
- hiring submarkets $\{W_s\}_s$
- Long-Term Contract $\mathcal{C}$

Directed Search

U_{Li}	U_{Hi}	U_{Le}	U_{He}

Transitions

One transition is realized:

- Endogenous: Contract $\mathcal{C}$
- Exogenous
  - firm dissolution s_F
  - worker separation s^W, λ_r
  - skill change λ^T, λ^E
  - productivity shock $F_z(z'|z)$

BGP

A stationary equilibrium consists of value functions $J(s), W(s), \Sigma(s)$, $\left\{w_i(s), v_i(s), \delta_i(s), W_i'(s)\right\}_{i\in\mathcal{S}}, \pi_e(s), a_\tau(s)$, unemployment values $\{\mathcal{U}_i\}_{i\in\mathcal{S}}$, a per-capita distribution $\Lambda(s)$, per-capita masses of active firms F and entrants F_e, per capita measures of employment and unemployment per each type $\{E_i, U_i\}_{i\in\mathcal{S}}$, such that:

- ***Optimality holds***.
- ***Distributions and measures per capita are stationary***.
- ***Free-entry holds***: $J^e = 0$

Back

Solve the model

- Define the Joint Surplus: $\Sigma(\underbrace{\vec{n}, z, \tau}_{s}) \equiv \underbrace{J(\vec{s})}_{\text{Firm's Value}} + \sum_i n_i \underbrace{W_i}_{\text{Worker's Value}}$

Proposition (Schaal [2017])

Solving the firm's problem is equivalent to maximize to joint surplus.

- Restricting to the class of Markov Perfect Equilibria

$$W_i(s) = \gamma \underbrace{\mathcal{U}_i}_{\text{Outside Option}} + (1-\gamma) \underbrace{\left[\Sigma(s) - \Sigma(s_i^-)\right]}_{\text{Contribution of } i \text{ to Surplus}} , \quad \forall i \in \mathcal{S},$$

- Wages are pinned down by promise keeping constraint holding with equality.
- We solve for stationary equilibrium per-capita.

Back

Potential Entrants

- Potential entrants' value:

$$J^e = \underbrace{-\kappa}_{\text{Entry Cost}} + \underbrace{p_0 \tilde{J}^e(\tau = 0) + (1 - p_0)\tilde{J}^e(\tau = 1)}_{\text{Expected Profits}}$$

  - Pay a flow cost to open a business: κ
  - Enter as frontier with probability p_0.

- Upon entry:

$$\tilde{J}^e(\tau) = \sum_z F^e(z) \left[\max_{\{W_i'\}} \sum_{i \in \mathcal{S}} \eta_i \left(W_i'(s_i^e) \right) J(s_i^e) \right], \ \forall i$$

  - Draw a **random productivity shock** from F_z^e
  - **Post one vacancy** for each type of worker, choosing the submarket W_i.

Back

Notation: Evolution of State Variables

Back

Remark: Given continuous time, only one state transition can happen simultaneously.

$$\vec{s}\,' \equiv (\vec{n}, z', \tau') \in \begin{cases} (n_{LL}+1, \vec{s}_-),\ (n_{LH}+1, \vec{s}_-),\ (n_{HL}+1, \vec{s}_-),\ (n_{HH}+1, \vec{s}_-) & \text{Hiring } i: \ \vec{s}_i^{\,+},\ \forall i \\ (n_{LL}-1, \vec{s}_-),\ (n_{LH}-1, \vec{s}_-),\ (n_{HL}-1, \vec{s}_-),\ (n_{HH}-1, \vec{s}_-) & \text{Firing } i: \ \vec{s}_i^{\,-},\ \forall i \\ (n_{LL}-1, n_{HL}+1, \vec{s}_-),\ (n_{LH}-1, n_{HH}+1, \vec{s}_-) & \text{Up-skilling}: \ \vec{s}_{\pi_e} \\ (n_{HL}-1, n_{LL}+1, \vec{s}_-),\ (n_{HH}-1, n_{LH}+1, \vec{s}_-) & \text{Obsolescence}: \ \vec{s}_{\lambda_T} \\ (n_{LL}-1, n_{LH}+1, \vec{s}_-),\ (n_{HL}-1, n_{HH}+1, \vec{s}_-) & \text{Experience}: \ \vec{s}_{\lambda_R} \\ (n_{LL}-1, \vec{s}_-),\ (n_{LH}-1, \vec{s}_-),\ (n_{HL}-1, \vec{s}_-),\ (n_{HH}-1, \vec{s}_-) & \text{Retirement}: \ \vec{s}_{\lambda_o} \\ (\tau', \vec{s}_-) & \text{Tech Change}: \ \vec{s}_{\tau'} \\ (z', \vec{s}_-) & \text{Prod. Shock}: \ \vec{s}_{z'} \end{cases}$$

Firm's Problem

Back

- Long-Term Contract: $\vec{\mathcal{C}} = \left\{\{w_i, v_i, \delta_i, W_i'(\vec{s})'\}_{i\in\{LL,LH,HL,HH\}},\ \pi_e,\ a_\tau\right\}$

$$
\begin{aligned}
\rho J(\vec{s}) = \max_{\vec{\mathcal{C}}} \Big\{ & y - C_e(\pi_e)(n_{LL}+n_{LH}) - C_\tau(a_\tau) + s^F\left(J^e - J(\vec{s})\right) \\
\textit{Workforce Costs} \quad & - \sum_i \left[w_i n_i + C_{\delta_i}(\delta_i) n_i + C_{v_i}(v_i) \right] \\
\textit{Hiring} \quad & + \sum_i v_i \eta_i \left(W_i'(\vec{s}_i^{\,+})\right)\left(J(\vec{s}_i^{\,+}) - J(\vec{s})\right) \Big\} \\
\textit{Separations} \quad & + \sum_i \left(\delta_i + s_i^W\right) n_i \left(J(\vec{s}_i^{\,-}) - J(\vec{s})\right) + \sum_i \lambda_r n_i \left(J(\vec{s}_{\lambda_o}) - J(\vec{s})\right) \\
\textit{Skill Change} \quad & + \sum_{i\in\{LL,LH\}} \pi_e \left(J(\vec{s}_{\pi_e}) - J(\vec{s})\right) + \sum_{i\in\{HL,HH\}} \lambda_T \left(J(\vec{s}_{\lambda_T}) - J(\vec{s})\right) + \sum_{i\in\{LL,HL\}} \lambda_P \left(J(\vec{s}_{\lambda_P}) - J(\vec{s})\right) \\
\textit{Tech Change} \quad & + \sum_{\tau'} \pi(\tau'|\tau, a)\left(J(\vec{s}_{\tau'}) - J(\vec{s})\right) \\
\textit{Prod. Shock} \quad & + \sum_{z'} F_z(z'|z)\left(J(\vec{s}_{z'}) - J(\vec{s})\right) \Big\} \qquad \text{s.t.} \quad \mathcal{W}_i(\vec{\mathcal{C}}) \geq W_i, \quad W_i' \geq \mathcal{U}_i, \quad \forall i
\end{aligned}
$$

Joint Surplus Maximization

$$(\rho+s^F)\Sigma(s) = \max_{\mathcal{C}^\Sigma}\Big\{ y - C_e(\pi_e)(n_{LL}+n_{LH}) - C_\tau(a_\tau) - \sum_i \Big[C_{\delta_i}(\delta_i)n_i + C_{v_i}(v_i)\Big]$$

Outside Option $\quad + \sum_i n_i \left(\delta_i + s_i^W + s^F\right)\mathcal{U}_i$

Utility to New Hires $\quad - \sum_i v_i\eta_i \left(W_i'(s_i^+)\right) W_i'(s_i^+)$

Hiring $\quad + \sum_i v_i\eta_i \left(W_i'(s_i^+)\right)\left(\Sigma(s_i^+) - \Sigma(s)\right)$

Separations $\quad + \sum_i \left(\delta_i + s_i^W\right) n_i \left(\Sigma(s_i^-) - \Sigma(s)\right) + \sum_i \lambda_r n_i \left(\Sigma(\vec{s}_{\lambda_o}) - \Sigma(\vec{s})\right)$

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Tech Change $\quad + \sum_{\tau'} \pi(\tau'|\tau.a_\tau)(\Sigma(s_{\tau'}) - \Sigma(s))$

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Joint Surplus

Joint Surplus Maximization

$$(\rho + s^F)\Sigma(s) = \max_{\mathcal{C}^\Sigma} \Bigg\{ y - C_e(\pi_e)(n_{LL} + n_{LH}) - C_\tau(a_\tau) - \sum_i \Big[C_{\delta_i}(\delta_i) n_i + C_{v_i}(v_i) \Big]$$

$$\textit{Outside Option} \qquad + \sum_i n_i \left(\delta_i + s_i^W + s^F \right) \mathcal{U}_i$$

$$\textit{Utility to New Hires} \qquad - \sum_i v_i \eta_i \left(W_i'(s_i^+) \right) W_i'(s_i^+)$$

$$\textit{Hiring} \qquad + \sum_i v_i \eta_i \left(W_i'(s_i^+) \right) \left(\Sigma(s_i^+) - \Sigma(s) \right)$$

$$\textit{Separations} \qquad + \sum_i \left(\delta_i + s_i^W \right) n_i \left(\Sigma(s_i^-) - \Sigma(s) \right) \; + \sum_i \lambda_r n_i \left(\Sigma(\vec{s}_{\lambda_o}) - \Sigma(\vec{s}) \right)$$

$$\textit{Skill Change} \qquad + \sum_{i \in \{LL, LH\}} \pi_e \left(\Sigma(\vec{s}_{\pi_e}) - \Sigma(\vec{s}) \right) + \sum_{i \in \{HL, HH\}} \lambda_T \left(\Sigma(\vec{s}_{\lambda_T}) - \Sigma(\vec{s}) \right) + \sum_{i \in \{LL, HL\}} \lambda_P \left(\Sigma(\vec{s}_{\lambda_P}) - \Sigma(\vec{s}) \right)$$

$$\textit{Tech Change} \qquad + \sum_{\tau'} \pi(\tau' | \tau . a_\tau)(\Sigma(s_{\tau'}) - \Sigma(s))$$

$$\textit{Prod. Shock} \qquad + \sum_{z'} F_z(z'|z)(\Sigma(s_{z'}) - J(s)) \Bigg\}$$

Joint Surplus

Aggregate Productivity Decomposition Back

Aggregate productivity

$$
\begin{aligned}
\frac{\mathcal{Y}}{E} = & \frac{1}{E}\sum_{s\in\mathcal{S}} y^*(s) f(s) \\
= & \frac{F}{E}\sum_{z,\tau}\frac{F_{z,\tau}}{F}\sum_{n} y^*(s)\tilde{f}_{z,\tau}(n) \\
= & \underbrace{\left(\frac{F}{E}\right)^{1-\nu}}_{\text{Firm Size}}\sum_{z,\tau}\underbrace{\frac{F_{z,\tau}}{F}}_{\text{Firm Selection}}\underbrace{\sum_{\hat{n},m} y^*_{z,\tau}(\hat{n},m) g_{z,\tau}(\hat{n},m)}_{\text{Workers Allocation}}
\end{aligned} \tag{1}
$$

where

$$
g_{z,\tau}(\hat{n},m) = \sum_{n}\tilde{f}_{z,\tau}(n)\mathbb{I}\left\{\frac{n}{E/F} = \hat{n} \ \ \& \ \ m_\tau = m\right\}
$$

Back

Productivity Decomposition

Output: $$y = zA_\tau l^\eta \Big[\underbrace{T^{\alpha_\tau} E^{1-\alpha_\tau}}_{m_\tau(T,E)}\Big]^\eta.$$

Aggregate Productivity

$$\frac{\mathcal{Y}}{E} = \underbrace{(\text{Avg Size})^{\eta-1}}_{\text{DRS}} \times \text{Avg (Normalized) Output} =$$

$$= (\text{Avg Size})^{\eta-1} \times \underbrace{\sum_{z,\tau} \pi(z,\tau)}_{\text{Productivity Selection}} \underbrace{\sum_{\hat{l}} \sum_{m} \text{Output}(\hat{l}, m) \times g_{z,\tau}(\hat{l}, m)}_{\text{Within-Productivity}}$$

$$= (\text{Avg Size})^{\eta-1} \times \underbrace{\sum_{z} \pi_z}_{z-\text{Selection}} \underbrace{\sum_{\tau} \pi_{\tau|z}}_{\text{Tech. Adoption}} z\lambda^{1-\tau} \underbrace{\sum_{\hat{l}} \hat{l}^\eta g_{z,\tau}(\hat{l})}_{\text{Size Allocation}} \underbrace{\sum_{T,E} T^{\alpha_\tau \eta} E^{(1-\alpha_\tau)\eta} g_{z,\tau}(T, E|\hat{l})}_{E+T \text{ Allocation}}$$

Back

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Back

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Back

Calibration

Production Function

Parameter	Value	Source
ρ_z	0.4000	
σ_z	0.3280	
η	0.8595	
α_1	0.5000	

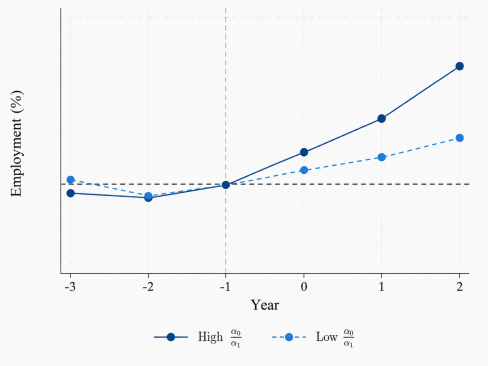
Other

Parameter	Value	Source
ρ	0.0125	
γ	0.5000	
ν_v	1.0001	
ν_δ	1.0001	
ν_e	1.0000	
p_{tr}	0.4000	
p_τ	0.3500	

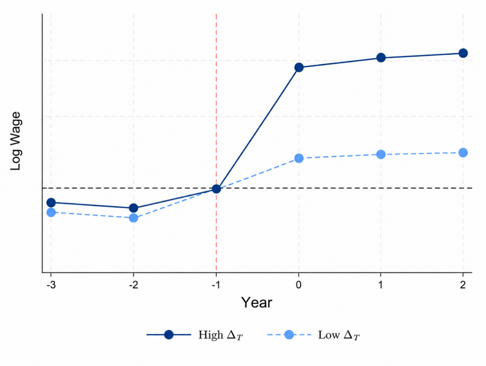
Notes:

Back

Model Simulated Event Study



(a) Employment (%), Young



(b) Log Wage, Young

Back